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Pancyclicity of OTIS (swapped) networks based on properties of the factor graph

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ABSTRACT

The plausibility of embedding cycles of different lengths in the graphs of a network (known as the pancyclicity property) has important applications in interconnection networks, parallel processing systems, and the implementation of a number of either computational or graph problems such as those used for finding storage schemes of logical data structures, layout of circuits in VLSI, etc. In this paper, we present the sufficient condition of the pancyclicity property of OTIS networks. The OTIS network (also referred to as two-level swapped network) is composed of n clones of an n-node original network constituting its clusters. It has received much attention due to its many favorable properties such as high degree of scalability, regularity, modularity, package-ability and high degree of algorithmic efficiency. Many properties of OTIS networks have been studied in the literature. In this work, we show that the OTIS networks have the pancyclicity property when the factor graph is Hamiltonian. More precisely, using a constructive method, we prove that if the factor graph G of an OTIS network contains cycles of length $\{3, 4, 5, l\}$, then all cycles of length $\{3, \ldots, l^2\}$, can be embedded in the OTIS-G network. This result resolves the open question posed and tracked in Day and AlAyyoub (2002) [2], Hoseiny Farahabady and Sarbazi Azad (2007) [4] and Shafiei et al. (2011) [14].

1. Introduction

The Optical Transpose Interconnection System (OTIS) networks, also known as swapped networks generate a wide class of high-performance scalable interconnection networks [11,19]. In this architecture, processors are divided into groups where electronic interconnects are used to connect processors within each group, while optical interconnects are used for inter-group communication. The processors of an N^2 processors within an OTIS system are partitioned into N groups of N processors [11]. It has been shown in [6] that when the number of processors in a group equals the number of groups, both the bandwidth and the power consumption in a group shaped network are optimized while both the system area and the volume of system are minimized. The OTIS-hypercube and OTIS-mesh are two of the most widely studied instances of the OTIS architecture [2,12,14]. A number of algorithms have been developed for OTIS networks, such as routing, selection, and data rearrangement and sorting [13,16], matrix multiplication [15], and broadcasting [2]. Many of topological properties of these systems such as node degree, diameter, β-cut, and bisection width are addressed in previous studies [2]. Furthermore in [4] it was proved that if G is a Hamiltonian-connected graph, so is the OTIS-G. In [12] the performance merits of the OTIS-hypercube and the effect of different structural and workload parameters

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on the overall performance are investigated. Their results reveal that the OTIS multi-computers are good candidates for interconnection networks of future generation of parallel computers.

In this paper, we fully solve the problem addressed and tracked in [2,4,14]. These works showed that if G contains a cycle of length l, then there exist both cycles of length l^2, $l^2 - l$ and $l^2 - kl$, $1 \leqslant k < l$, in OTIS-G, respectively. In [4], it was also demonstrated that if there exists a Hamiltonian path between every two arbitrary nodes of graph G (i.e. G is Hamiltonian-connected), then all cycles of lengths $\{7, 8, \ldots, |G|^2\}$ can be constructed in the OTIS-G.

The pancyclicity of a specified system is an essential feature in determining whether the system topology is able to imitate rings of different length. It is identified that a lot of standard parallel algorithms have a circle construction [7]. To perform a circle-structure corresponding algorithm on a particular system efficiently, the processes of the parallel algorithm need to be plotted to the nodes of the interconnection network in the system in such a way that any two processes that are adjacent in the circle structure are mapped to two adjacent nodes of the network. Hence, a well-organized mapping requires that the system owns a cycle of a specified length. In addition, ring structure may be employed as control structures for distributed systems. Regarding these wide range of applications, it is preferred that an interconnection network be pancyclic. A graph G is pancyclic if G contains a cycle of length l for each integer l with $3 \leqslant l \leqslant V(G)$.

Nevertheless, the techniques of build up Hamiltonian cycles or the methodology of pancyclicity verification in different networks are not similar and merely is appropriate for their own particular topology. As there is not any unique outlook to these alternatives, it is hard to extend the proof consequences of one topology to another one even in the same family.

Here, we prove that if the factor graph G contains a cycle of length l, $l \leqslant N$ (N being the size of the network, i.e. $|V(G)| = N$), we can form all cycles of lengths $\{7, \ldots, l^2\}$, in the OTIS-G. In addition, we present a sufficient condition in the factor graph for the pancyclicity of OTIS networks. The rest of the paper is organized as follows. In Section 2, some definition OTIS networks is given. Section 3 presents our results. Finally Section 4 concludes the paper.

2. The OTIS network

Interested reader is referred to [2] for an in-depth account of basic concepts and properties of the OTIS networks such as topology, routing algorithms, broadcasting, embedding of graphs, etc.

Definition 1. Let $G = (V(G), E(G))$ be an undirected graph, which called the factor graph of OTIS-G, then the OTIS-$G = (V(OT), E(OT))$ graph is an undirected graph where:

$$V(OT) = \{\langle g, p\rangle \mid g, p \in V(G)\}$$

and

$$E(OT) = \{(\langle g, p_1\rangle, \langle g, p_2\rangle) \mid g \in V(G),\ p_1, p_2 \in E(G)\} \cup \{(\langle g, p\rangle, \langle p, g\rangle) \mid g, p \in V(G),\ g \neq p\}$$

OTIS-G is composed of exactly $|V_G| = N$ copies of graph G, each of which called a group and denoted as $G_1, G_2, \ldots, G_N$. A node $\langle g, p\rangle$ in OTIS-G corresponds to node p in group G_g. An intra-group edge of the form $(\langle g, p_1\rangle, \langle g, p_2\rangle)$ corresponds to an electronic link, while an inter-group edge of the form $(\langle g, p\rangle, \langle p, g\rangle)$ corresponds to a transpose (optical) link.

In [20] OTIS is used to realize interconnection networks such as hypercube, 4-D mesh, mesh of trees and butterfly for multiprocessor systems, also show some of wires in these networks can replace with transmitter and receiver by OTIS architecture, and conclude interesting result in speed, power consumption and space reduction.

There are several interesting features; especially when the number of groups equals the number of processors in each group; as illustrated in [2]. Also there were several basic operations and topological properties were developed in OTIS network, including optimal routing, data sum, size, degree, diameter [2,16]. In addition, a large set of problems in OTIS networks are solved include routing [12,13], load balancing [10], selection [13], sorting [13,16], matrix multiplication [15], polynomial interpolation and polynomial root finding [5,9] and image processing [17].

3. Pancyclicity of OTIS networks

Pancyclicity in a network is an important issue which enables that networks to exploit many algorithms designed for cycles. The pancyclicity of a large set of networks, like crossed cubes, Möbius cube, k-Ary n-Cube, WK-Recursive network and OTIS-mesh network are proven in [3,8,14,18]. Indeed similar properties of network can be defined by applying a few changes in the definition of pancyclicity. These properties are issue of great importance and interests. For example, a new work conducted recently deals with the panconnectivity and edge-pancyclicity of k-Ary n-Cube graph reported in [8]. As well, there are several general results deal with the pancyclicity property of graph. Bondy in [1] has proved that if the minimum degree of a network of size N is $N/2$, then it is a pancyclic graph. In [1], it is shown that every Hamiltonian network G of size N with a minimum number of edges $N^2/4$ is pancyclic. Zhang [21] has shown that if G is a Hamiltonian network with N vertices of maximum and minimum degree $\Delta(G)$ and $\delta(G)$, then it is pancyclic if $\Delta(G) + \delta(G) \geqslant N$.

None of the above cases is applicable to the OTIS-G network. In this paper, we deal directly with this particular network to prove that any OTIS-G is pancyclic.

Definition 2. A Hamiltonian cycle (path) is a cycle (path) containing every vertex of G. A graph is Hamiltonian if it contains a Hamiltonian cycle.

Definition 3. A graph G is called pancyclic if it contains every *k-cycle* for $3 \leqslant k \leqslant V(G)$. More precisely, for a graph $G = (V, E)$ and a given set $\Sigma = \{3, 4, \ldots, V(G)\}$, graph G is called to be *Σ-pancyclic* if G contains all cycles of length

$\sigma \in \Sigma$, comparably, G is said to be $\bar{\Sigma}$*-pancyclic* if G contains all cycles of length $\sigma \in \{3, 4, \ldots, V(G)\} - \Sigma$ [4].

In this paper, we prove that if there exists an *l-cycle* in the factor graph G, then all cycles of length 7 to l^2 can be formed in OTIS-G. Furthermore, if there exist cycles of lengths $\{3, 4, 5, N\}$ in the factor graph G, then OTIS-G is pancyclic.

Theorem 1. *If there exists an L-cycle in G, then there exists a cycle of length $L^2 - L$ in* OTIS-G [2].

Theorem 2. *If graph G contains an L-cycle then there exists an L^2-cycle in* OTIS-G [4].

Theorem 3. *If a factor graph G has a cycle of length l, then* OTIS-G *has all cycles of lengths* 7 *to* $l^2 - 1$.

Proof. Take an arbitrary graph G with a cycle of length l, $l \geqslant 8$.

Remark. For the special cases when $4 \leqslant l \leqslant 7$, please refer to Appendix A for further details.

Without loss of generality we can assume the cycle is of the form $C_l = \langle \gamma_1, \gamma_2, \ldots, \gamma_l, \gamma_1 \rangle$, that contains edges of the form (γ_v, γ_{v+1}) (for $1 \leqslant v < l$) and edge (γ_l, γ_1). It is worthy to mention that if C_l is of the another form like $\langle \gamma_{f(1)}, \gamma_{f(2)}, \ldots, \gamma_{f(l)}, \gamma_{f(1)} \rangle$ for some function $f : \{1, 2, \ldots, l\} \to \{1, 2, \ldots, V(G)\}$, we could replace the value of i with $f(i)$ and the proof goes true since we could use edges of the form $(\langle \gamma_{f(i)}, \gamma_{f(j)} \rangle, \langle \gamma_{f(i)}, \gamma_{f(j+1)} \rangle)$, $(\langle \gamma_{f(i)}, \gamma_{f(l)} \rangle, \langle \gamma_{f(i)}, \gamma_{f(1)} \rangle)$ and $(\langle \gamma_{f(i)}, \gamma_{f(j)} \rangle, \langle \gamma_{f(j)}, \gamma_{f(i)} \rangle)$ instead of edges of the form $(\langle \gamma_i, \gamma_j \rangle, \langle \gamma_i, \gamma_{j+1} \rangle)$, $(\langle \gamma_i, \gamma_l \rangle, \langle \gamma_i, \gamma_1 \rangle)$ and $(\langle \gamma_i, \gamma_j \rangle, \langle \gamma_j, \gamma_i \rangle)$, respectively, without the need to change any of the other symbols.

Let G_i denote the i-th group of OTIS-G, and let $(\langle \gamma_i, \gamma_j \rangle)$ denote the j-th node within the group G_i.

To form the cycle of length q $(7 \leqslant q \leqslant l^2)$, we consider the three separated cases:

Case 1. $q \leqslant 3l$.

To form the *q-cycle* within OTIS-G, where $q = 2x + 1$, $7 \leqslant q \leqslant 2l + 1$, the following construction method can be used (note that these cycles only use three different groups in OTIS-G, namely G_1, G_2 and G_x):

$$C_{q=2x+1}\colon \big(\langle \gamma_1, \gamma_2 \rangle, \langle \gamma_1, \gamma_3 \rangle, \ldots, \langle \gamma_1, \gamma_x \rangle \langle \gamma_x, \gamma_1 \rangle, \langle \gamma_x, \gamma_2 \rangle \| \langle \gamma_2, \gamma_x \rangle, \ldots, \langle \gamma_2, \gamma_2 \rangle, \langle \gamma_2, \gamma_1 \rangle \| \langle \gamma_1, \gamma_2 \rangle \big)$$

Symbol $\|$ represents an intra-group link within the OTIS-G.

We use the following schema to build up the cycle of length q, where $q = 2x + l - 1$, $l + 5 \leqslant q \leqslant 3l - 1$, and l is the length of given cycle in the factor graph (note that this cycle passes only through the vertices of groups G_1, G_2 and G_x within the OTIS-G):

$$C_{q=2x+l-1}\colon \big(\langle \gamma_1, \gamma_2 \rangle, \langle \gamma_1, \gamma_3 \rangle, \ldots, \langle \gamma_1, \gamma_x \rangle \| \langle \gamma_x, \gamma_1 \rangle, \langle \gamma_x, \gamma_l \rangle, \langle \gamma_x, \gamma_{l-1} \rangle, \ldots, \langle \gamma_x, \gamma_2 \rangle \| \langle \gamma_2, \gamma_x \rangle, \langle \gamma_2, \gamma_{x-1} \rangle, \ldots, \langle \gamma_2, \gamma_2 \rangle, \langle \gamma_2, \gamma_1 \rangle \| \langle \gamma_1, \gamma_2 \rangle \big)$$

To form the cycle of length q, where $q = 6 + 2((y - x + l) \bmod l)$, $8 \leqslant q \leqslant 2l + 4$, $3 \leqslant x, y \leqslant l$, the following schema may be applied (note that this type of cycle passes through the vertexes of groups G_1, G_2, G_x and G_y within the OTIS-G):

$$C_{q=6+2(y-x)}\colon \big(\langle \gamma_1, \gamma_x \rangle, \langle \gamma_1, \gamma_{x+1} \rangle, \ldots, \langle \gamma_1, \gamma_{y-1} \rangle, \langle \gamma_1, \gamma_y \rangle \| \langle \gamma_y, \gamma_1 \rangle, \langle \gamma_y, \gamma_2 \rangle \| \langle \gamma_2, \gamma_y \rangle, \langle \gamma_2, \gamma_{y-1} \rangle, \ldots \rangle, \langle \gamma_2, \gamma_x \rangle \| \langle \gamma_x, \gamma_2 \rangle, \langle \gamma_x, \gamma_1 \rangle \| \langle \gamma_1, \gamma_x \rangle \big)$$

In a similar manner to the one presented above, we can construct a cycle C of length q, where:

$$q = l + 4 + 2(y - x); \quad l + 6 \leqslant q \leqslant 3l + 2, \quad 3 \leqslant x, y \leqslant l$$

It is worth to mention that this cycle passes through the vertices of only four groups, namely G_1, G_2, G_x and G_y:

$$C_{q=l+4+2(y-x)}\colon \big(\langle \gamma_1, \gamma_x \rangle, \langle \gamma_1, \gamma_{x+1} \rangle, \ldots, \langle \gamma_1, \gamma_{y-1} \rangle, \langle \gamma_1, \gamma_y \rangle \| \langle \gamma_y, \gamma_1 \rangle, \langle \gamma_y, \gamma_2 \rangle \| \langle \gamma_2, \gamma_y \rangle, \langle \gamma_2, \gamma_{y-1} \rangle, \ldots \rangle, \langle \gamma_2, \gamma_x \rangle \| \langle \gamma_x, \gamma_2 \rangle, \langle \gamma_x, \gamma_3 \rangle, \ldots, \langle \gamma_x, \gamma_l \rangle, \langle \gamma_x, \gamma_1 \rangle \| \langle \gamma_1, \gamma_x \rangle \big)$$

Altogether, we have found all cycles of length 7 to $3l$ in the OTIS-G.

Case 2. When either $3l + 1 \leqslant q \leqslant l^2 - 5$ or $q = l^2 - 3$. We can always take the following expression that contains the new parameters ω, s and δ to write q, $l \geqslant 8$:

$$q = (\omega + 3) * (l + 1) - 2s - \delta$$

where $1 \leqslant \omega \leqslant l - 4$; $1 \leqslant s \leqslant l - 4$; $\delta \in \{0, 3, 6\}$. A simple program can easily calculate the value of parameters ω, s and δ to make the true value of q.

To deal with this case, we define three different sets of groups within the OTIS-G, namely *pivot* set, *fragmentary* set and *incremental* set. Each of these three sets consists of different nodes and serves a different role to form the desired cycle within the OTIS-G. More precisely, we use *incremental* set to constitute a path of length $\omega * (l + 1)$, nodes, *pivot* set to create a path of length l, and *fragmentary* set to make a path of length $2(l - s) + 3 - \delta$. Altogether, by connecting these three paths, we find a cycle with total length of:

$$\begin{aligned} &(l + 1) * \omega + l + 2 * (l - s) + 3 - \delta \\ &\quad = (\omega + 3) * (l + 1) - 2s - \delta = q \end{aligned}$$

In the rest of this section, we illustrate how to make these three sets and connect them.

The *incremental* set contains the groups of $G_1, G_2, \ldots, G_{l-4}$. Each group of this set may or may not be used to construct the desired *q-cycle*. If we use one of them, it serves to contribute to form the *q-cycle* by sharing $l + 1$ edges. More specifically it shares $l - 1$ edges belong to that group and 2 intra-group edges which connect this group to two corresponding nodes within either the *pivot* set. There is also a special group called G_S. This group connects the *fragmentary* section to the rest of the *q-cycle* by sharing edges of the form $(\langle \gamma_S, \gamma_{l-2} \rangle, \langle \gamma_{l-2}, \gamma_S \rangle)$ and

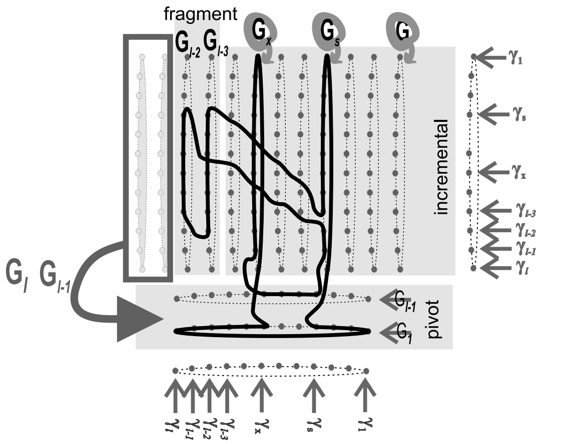

Fig. 1. How to construct three groups of *pivot* nodes, *fragmentary* nodes and *incremental* nodes.

$(\langle\gamma_S,\gamma_{l-3}\rangle,\langle\gamma_{l-3},\gamma_S\rangle)$. Hence, it can be easily inferred that G_S shares only $l-2$ inter-group edges. For example, in Fig. 1, groups namely G_x and G_S are selected to be in *incremental* set. Group G_x contributes to share both its own $l-1$ inter-group edges and also edges of the form $(\langle\gamma_x,\gamma_{l-1}\rangle,\langle\gamma_{l-1},\gamma_x\rangle)$ and $(\langle\gamma_x,\gamma_l\rangle,\langle\gamma_l,\gamma_x\rangle)$ to construct the desired *q-cycle*.

Pivot set consists of groups G_{l-1} and G_l which contributes l edge to the *q-cycle*. The role of this set is to make connections between the paths in groups of *incremental* set. More precisely, each group of G_x in *incremental* set, connects to the nodes $(\langle\gamma_{l-1},\gamma_x\rangle,\langle\gamma_l,\gamma_x\rangle)$ and within *pivot* set via intra-group edges. *Pivot* set shares exactly one edge of the form $(\langle\gamma_{l-1},\gamma_v\rangle,\langle\gamma_{l-1},\gamma_{v-1 \bmod l}\rangle)$ or $(\langle\gamma_l,\gamma_v\rangle,\langle\gamma_l,\gamma_{v-1 \bmod l}\rangle)$, where $1\leqslant v\leqslant l-1$. Let ω be the number of groups in the incremental set and $\alpha_1,\alpha_2,\dots,\alpha_\omega$ be the group numbers were selected to make the incremental set, where $\alpha_1>\alpha_2>\dots>\alpha_\omega$. The *pivot* set contains all the edges in a path of the form $(\langle\gamma_{l-1},\alpha_i\rangle,\langle\gamma_{l-1},\alpha_i-1\rangle,\langle\gamma_{l-1},\alpha_i-2\rangle,\dots,\langle\gamma_{l-1},\alpha_{i+1}\rangle)$, $1\leqslant i<\omega$, if i is odd. On the contrary, it contains the edges in a path of the form $(\langle\gamma_l,\alpha_i\rangle,\langle\gamma_l,\alpha_i-1\rangle,\langle\gamma_l,\alpha_i-2\rangle,\dots,\langle\gamma_l,\alpha_{i+1}\rangle)$, $1\leqslant i<\omega$, if i is even. Moreover, if ω is odd, a path in the form of $(\langle\gamma_{l-1},\alpha_\omega\rangle,\langle\gamma_{l-1},\alpha_\omega-1\rangle,\langle\gamma_{l-1},\alpha_\omega-2\rangle,\dots,\langle\gamma_{l-1},\alpha_1\rangle,\langle\gamma_{l-1},\alpha_l\rangle\|\langle\gamma_l,\alpha_{l-1}\rangle,\langle\gamma_l,\alpha_{l-2}\rangle,\dots,\langle\gamma_l,\alpha_1\rangle)$ is belonging to the *pivot* set. On the other hand, if ω is even, *pivot* set contains a path like $(\langle\gamma_l,\alpha_\omega\rangle,\langle\gamma_l,\alpha_\omega-1\rangle,\dots,\langle\gamma_l,1\rangle,\langle\gamma_l,l\rangle,\langle\gamma_l,l-1\rangle,\langle\gamma_l,l-2\rangle,\dots,\langle\gamma_l,\alpha_1\rangle)$.

The *fragmentary* set consists of two specified groups of G_{l-2} and G_{l-3} which plays the role of completing the length of the *q-cycle* remained already by compiling both *incremental* and *pivot* sets. More precisely, *fragmentary* set is used to create some paths of total lengths $2(l-s)+3-\delta$, $1\leqslant s\leqslant l-4$; $\delta\in\{0,3,6\}$ which connected to the group G_s within the *incremental* set, yielding the desired *q-cycle*. If $\delta=6$, the path in the *fragmentary* set like in the form of:

$$P_{frag}\colon \big(\langle\gamma_s,\gamma_{l-2}\rangle\|\langle\gamma_{l-2},\gamma_s\rangle,\langle\gamma_{l-2},\gamma_{s+1}\rangle,\dots, \langle\gamma_{l-2},\gamma_{l-3}\rangle\|\langle\gamma_{l-3},\gamma_{l-2}\rangle,\langle\gamma_{l-3},\gamma_{l-3}\rangle,\dots, \langle\gamma_{l-3},\gamma_s\rangle\|\langle\gamma_s,\gamma_{l-3}\rangle\big)$$

In addition, if $\delta=3$ or $\delta=0$, another path is added to the previous cycle to create the q-cycle as follows:

$$P_{\delta=3}\colon \big(\langle\gamma_{l-1},\gamma_{l-2}\rangle\|\langle\gamma_{l-2},\gamma_{l-1}\rangle,\langle\gamma_{l-2},\gamma_l\rangle\|\langle\gamma_l,\gamma_{l-2}\rangle\big)$$

$$P_{\delta=0}\colon \big(\langle\gamma_{l-1},\gamma_{l-3}\rangle\|\langle\gamma_{l-3},\gamma_{l-1}\rangle,\langle\gamma_{l-3},\gamma_l\rangle\|\langle\gamma_l,\gamma_{l-3}\rangle\big)$$

Case 3. When $q\in\{l^2-4,l^2-2,l^2-1\}$. Slightly modifying the method used to prove Case 2, one can solve the remaining three cases. Let $u<v$. The sequence $\langle\gamma_u,\gamma_{u+1},\gamma_{u+2},\dots,\gamma_{v-1},\gamma_v\rangle$ forms a path between two arbitrary nodes of u and v in factor graph G, group γ_i, can be shown by $P_{\gamma_i}(\overrightarrow{u:v})$. On the other hand, we could form another path between these two nodes in group γ_i as the following which we have showed by $P_{\gamma_i}(\overleftarrow{u:v})$:

$$P_{\gamma_i}(\overleftarrow{u:v})\colon \langle\gamma_u,\gamma_{u-1},\dots,\gamma_2,\gamma_1,\gamma_l,\dots,\gamma_{v+1},\gamma_v\rangle$$

In a similar way, if $u>v$, we could define two mentioned paths as follows:

$$P_{\gamma_i}(\overrightarrow{u:v})\colon \langle\gamma_u,\gamma_{u+1},\dots,\gamma_{l-1},\gamma_l,\gamma_1,\dots,\gamma_{v-1},\gamma_v\rangle$$

$$P_{\gamma_i}(\overleftarrow{u:v})\colon \langle\gamma_u,\gamma_{u-1},\dots,\gamma_{v+1},\gamma_v\rangle$$

Now, we can express our method to build three cycles of length l^2-4, l^2-2, and l^2-1.

Subcase 3.1. When l is *even*:

$$\begin{aligned} C_{l^2-4}\colon \big(&P_{\gamma_2}(\overleftarrow{l-1:l-3})\|P_{\gamma_{l-3}}(\overrightarrow{2:l-2})\|\\ &P_{\gamma_{l-2}}(\overrightarrow{l-3:l-1})\|\langle\gamma_{l-1},\gamma_{l-2}\rangle,\langle\gamma_{l-1},\gamma_{l-3}\rangle\|\\ &\langle\gamma_{l-3},\gamma_{l-1}\rangle,\langle\gamma_{l-3},\gamma_l\rangle\|\langle\gamma_l,\gamma_{l-3}\rangle,\langle\gamma_l,\gamma_{l-4}\rangle\|\\ &\langle\gamma_{l-4},\gamma_l\rangle,\langle\gamma_{l-4},\gamma_{l-1}\rangle\|\langle\gamma_{l-1},\gamma_{l-4}\rangle,\langle\gamma_{l-1},\gamma_{l-5}\rangle\|\\ &P_{\gamma_{l-5}}(\overleftarrow{l-1:l})\|\langle\gamma_l,\gamma_{l-5}\rangle,\langle\gamma_l,\gamma_{l-6}\rangle\|\\ &P_{\gamma_{l-6}}(\overrightarrow{l:l-1})\|\cdots P_{\gamma_2}(\overrightarrow{l:l-4})\|P_{\gamma_{l-4}}(\overrightarrow{2:l-2})\|\\ &P_{\gamma_{l-2}}(\overleftarrow{l-4:l})\|P_{\gamma_l}(\overrightarrow{l-2:1})\|P_{\gamma_1}(\overrightarrow{l:1-1})\|\\ &\langle\gamma_{l-1},\gamma_1\rangle,\langle\gamma_{l-1},\gamma_2\rangle\big) \end{aligned}$$

$$\begin{aligned} C_{l^2-2}\colon \big(&P_{\gamma_1}(\overrightarrow{l:l-1})\|\langle\gamma_{l-1},\gamma_1\rangle,\langle\gamma_{l-1},\gamma_2\rangle\|\\ &P_{\gamma_2}(\overleftarrow{l-1:l})\|\langle\gamma_l,\gamma_2\rangle,\langle\gamma_l,\gamma_3\rangle\|P_{\gamma_3}(\overrightarrow{l:l-1})\\ &\cdots\|P_{\gamma_{l-2}}(\overleftarrow{l-1:l})\|\langle\gamma_l,\gamma_{l-2}\rangle,\langle\gamma_l,\gamma_{l-1}\rangle,\\ &\langle\gamma_l,\gamma_l\rangle,\langle\gamma_l,\gamma_1\rangle\|\langle\gamma_1,\gamma_l\rangle\big) \end{aligned}$$

$$\begin{aligned} C_{l^2-1}\colon \big(&P_{\gamma_2}(\overrightarrow{l:l-1})\|\langle\gamma_{l-1},\gamma_2\rangle,\langle\gamma_{l-1},\gamma_3\rangle\|\\ &P_{\gamma_3}(\overleftarrow{l-1:l})\|\langle\gamma_l,\gamma_3\rangle,\langle\gamma_l,\gamma_4\rangle\|P_{\gamma_4}(\overrightarrow{l:l-1})\\ &\cdots\|P_{\gamma_{l-5}}(\overleftarrow{l-1:l})\|\langle\gamma_l,\gamma_{l-5}\rangle,\langle\gamma_l,\gamma_{l-4}\rangle\|\\ &\langle\gamma_{l-4},\gamma_l\rangle,\langle\gamma_{l-4},\gamma_{l-1}\rangle\|\langle\gamma_{l-1},\gamma_{l-4}\rangle,\langle\gamma_{l-1},\gamma_{l-3}\rangle\|\\ &\langle\gamma_{l-3},\gamma_{l-1}\rangle,\langle\gamma_{l-3},\gamma_l\rangle\|\langle\gamma_l,\gamma_{l-3}\rangle,\langle\gamma_l,\gamma_{l-2}\rangle,\\ &\|P_{\gamma_{l-2}}(\overrightarrow{l:l-4})\|P_{\gamma_{l-4}}(\overleftarrow{l-2:1})\|P_{\gamma_1}(\overleftarrow{l-4:l-3})\|\\ &P_{\gamma_{l-3}}(\overrightarrow{1:l-2})\|P_{\gamma_{l-2}}(\overrightarrow{l-3:l-1})\|\\ &P_{\gamma_{l-1}}(\overleftarrow{l-2:l})\|\langle\gamma_l,\gamma_{l-1}\rangle,\langle\gamma_l,\gamma_l\rangle,\langle\gamma_l,\gamma_1\rangle,\langle\gamma_l,\gamma_2\rangle\big) \end{aligned}$$

Subcase 3.2. When l is *odd*:

$$
\begin{aligned}
C_{l^2-4}\colon \big(& P_{\gamma_2}(\overleftarrow{l-1:l-4}) \| P_{\gamma_{l-4}}(\overrightarrow{2:l-3}) \| \\
& P_{\gamma_{l-3}}(\overrightarrow{l-4:l-1}) \| \langle \gamma_{l-1}, \gamma_{l-3} \rangle, \langle \gamma_{l-1}, \gamma_{l-4} \rangle \| \\
& P_{\gamma_{l-4}}(\overrightarrow{l-1:l}) \| \langle \gamma_l, \gamma_{l-4} \rangle, \langle \gamma_l, \gamma_{l-3} \rangle \| \\
& P_{\gamma_{l-3}}(\overrightarrow{l:l-1}) \| \langle \gamma_{l-1}, \gamma_{l-5} \rangle, \langle \gamma_{l-1}, \gamma_{l-6} \rangle \| \\
& P_{\gamma_{l-6}}(\overleftarrow{l-1:l}) \| \langle \gamma_l, \gamma_{l-6} \rangle, \langle \gamma_l, \gamma_{l-7} \rangle \| \cdots \\
& P_{\gamma_3}(\overleftarrow{l-1:l}) \| \langle \gamma_l, \gamma_3 \rangle, \langle \gamma_l, \gamma_2 \rangle \| P_{\gamma_2}(\overrightarrow{l:l-5}) \| \\
& P_{\gamma_{l-5}}(\overrightarrow{2:l-3}) \| P_{\gamma_{l-3}}(\overleftarrow{l-5:l}) \| \langle \gamma_l, \gamma_{l-3} \rangle, \langle \gamma_l, \gamma_{l-2} \rangle \| \\
& P_{\gamma_{l-2}}(\overrightarrow{l:l-1}) \| P_{\gamma_{l-1}}(\overrightarrow{l-2:l}) \| \langle \gamma_l, \gamma_{l-1} \rangle, \langle \gamma_l, \gamma_1 \rangle, \\
& \langle \gamma_l, \gamma_l \rangle \| P_{\gamma_1}(\overrightarrow{l:l-1}) \| \langle \gamma_{l-1}, \gamma_1 \rangle, \langle \gamma_{l-1}, \gamma_2 \rangle \big)
\end{aligned}
$$

$$
\begin{aligned}
C_{l^2-2}\colon \big(& P_{\gamma_1}(\overleftarrow{l-1:l}) \| \langle \gamma_l, \gamma_1 \rangle, \langle \gamma_l, \gamma_2 \rangle \| \\
& P_{\gamma_2}(\overrightarrow{l:l-1}) \| \langle \gamma_{l-1}, \gamma_2 \rangle, \langle \gamma_{l-1}, \gamma_3 \rangle \| \\
& P_{\gamma_3}(\overleftarrow{l-1:l}) \cdots \| P_{\gamma_{l-2}}(\overleftarrow{l-1:l}) \| \langle \gamma_l, \gamma_{l-2} \rangle, \langle \gamma_l, \gamma_{l-1} \rangle \| \\
& \langle \gamma_{l-1}, \gamma_l \rangle, \langle \gamma_{l-1}, \gamma_1 \rangle \big)
\end{aligned}
$$

$$
\begin{aligned}
C_{l^2-1}\colon \big(& P_{\gamma_2}(\overleftarrow{l-1:l}) \| \langle \gamma_l, \gamma_2 \rangle, \langle \gamma_l, \gamma_3 \rangle \| \\
& P_{\gamma_3}(\overrightarrow{l:l-1}) \| \cdots \| P_{\gamma_{l-4}}(\overrightarrow{l:l-1}) \| \\
& P_{\gamma_{l-3}}(\overrightarrow{l-1:l}) \| \langle \gamma_l, \gamma_{l-3} \rangle, \langle \gamma_l, \gamma_{l-2} \rangle \| \\
& P_{\gamma_{l-2}}(\overleftarrow{l:l-1}) \| \langle \gamma_{l-1}, \gamma_{l-2} \rangle, \\
& \langle \gamma_{l-1}, \gamma_{l-1} \rangle, \langle \gamma_{l-1}, \gamma_l \rangle \| \langle \gamma_l, \gamma_{l-1} \rangle, \langle \gamma_l, \gamma_l \rangle \| \\
& P_{\gamma_1}(\overrightarrow{l:l-3}) \| P_{\gamma_{l-3}}(\overrightarrow{1:l-2}) \| P_{\gamma_{l-2}}(\overleftarrow{l-3:1}) \| \\
& P_{\gamma_1}(\overrightarrow{l-2:l-1}) \| \langle \gamma_{l-1}, \gamma_1 \rangle, \langle \gamma_{l-1}, \gamma_2 \rangle \big) \qquad \square
\end{aligned}
$$

Theorem 4. *The sufficient condition for pancyclicity of* OTIS-*G is that the factor graph G contains all cycles of length* $\{3, 4, 5, |V(G)|\}$.

Proof. The proof here is fairly straightforward using Theorems 1, 2, 3 and special cases which describe in Appendix A. When factor graph G has cycle of length $|V(G)|$, all cycles of length 7 to $|V(G)|^2$ in OTIS-G can be made. Furthermore, using cycle of length 3 with vertex $\langle \gamma_1, \gamma_2, \gamma_3 \rangle$, a cycle of length 6 can be made as follows:

$$
\begin{aligned}
C_6\colon \big(& \langle \gamma_1, \gamma_2 \rangle, \langle \gamma_1, \gamma_3 \rangle \| \langle \gamma_3, \gamma_1 \rangle, \langle \gamma_3, \gamma_2 \rangle \| \\
& \langle \gamma_2, \gamma_3 \rangle, \langle \gamma_2, \gamma_1 \rangle \big)
\end{aligned}
$$

Borrowing cycles of length 3, 4 and 5 in one of the factor graphs, we have all cycles of length 3 to $|V(G)|^2$ in OTIS-G. $\square$

4. Conclusion

In this paper, we investigated one of the most important properties of OTIS network which is a good candidate for on-chip networking of current multicore and CMP systems-on-chip.

Our main result proves the conjecture expressed in [2, 4] that an OTIS-G network has every cycle of length 7 to l^2, if the factor graph G contains an *l-cycle*. Moreover, our result provides a sufficient condition for the purpose of pancyclicity in the OTIS-G network, which is an important property that eases development of some useful parallel algorithms. Evaluating the performance and power consumption of on-chip networks based on OTIS topologies is left for future work.

Table 1
Cycles of particular length for special cases in OTIS-G.

$\|V(G)\|$	$\|C\|$	Actual cycle
4	13	$(\langle \gamma_1, \gamma_4 \rangle, \langle \gamma_1, \gamma_1 \rangle, \langle \gamma_1, \gamma_3 \rangle \| \langle \gamma_3, \gamma_1 \rangle, \langle \gamma_3, \gamma_3 \rangle, \langle \gamma_3, \gamma_2 \rangle \| \langle \gamma_2, \gamma_3 \rangle, \langle \gamma_2, \gamma_2 \rangle, \langle \gamma_2, \gamma_1 \rangle, \langle \gamma_2, \gamma_4 \rangle \| \langle \gamma_4, \gamma_2 \rangle, \langle \gamma_4, \gamma_3 \rangle, \langle \gamma_4, \gamma_1 \rangle)$
4	14	$(\langle \gamma_1, \gamma_3 \rangle, \langle \gamma_1, \gamma_2 \rangle \| \langle \gamma_2, \gamma_1 \rangle, \langle \gamma_2, \gamma_2 \rangle, \langle \gamma_2, \gamma_3 \rangle, \langle \gamma_2, \gamma_4 \rangle \| \langle \gamma_4, \gamma_2 \rangle, \langle \gamma_4, \gamma_1 \rangle, \langle \gamma_4, \gamma_4 \rangle, \langle \gamma_4, \gamma_3 \rangle \| \langle \gamma_3, \gamma_4 \rangle, \langle \gamma_3, \gamma_3 \rangle, \langle \gamma_3, \gamma_2 \rangle, \langle \gamma_3, \gamma_1 \rangle)$
5	17	$(\langle \gamma_1, \gamma_5 \rangle, \langle \gamma_1, \gamma_4 \rangle, \langle \gamma_1, \gamma_3 \rangle \| \langle \gamma_3, \gamma_1 \rangle, \langle \gamma_3, \gamma_5 \rangle, \langle \gamma_3, \gamma_4 \rangle, \langle \gamma_3, \gamma_3 \rangle, \langle \gamma_3, \gamma_2 \rangle \| \langle \gamma_2, \gamma_3 \rangle, \langle \gamma_2, \gamma_4 \rangle \| \langle \gamma_4, \gamma_2 \rangle, \langle \gamma_4, \gamma_3 \rangle, \langle \gamma_4, \gamma_4 \rangle, \langle \gamma_4, \gamma_5 \rangle \| \langle \gamma_5, \gamma_4 \rangle, \langle \gamma_5, \gamma_5 \rangle, \langle \gamma_5, \gamma_1 \rangle)$
5	18	$(\langle \gamma_1, \gamma_3 \rangle, \langle \gamma_1, \gamma_2 \rangle, \langle \gamma_1, \gamma_1 \rangle, \langle \gamma_1, \gamma_5 \rangle \| \langle \gamma_5, \gamma_1 \rangle, \langle \gamma_5, \gamma_5 \rangle, \langle \gamma_5, \gamma_4 \rangle, \langle \gamma_5, \gamma_3 \rangle, \langle \gamma_5, \gamma_2 \rangle \| \langle \gamma_2, \gamma_5 \rangle, \langle \gamma_2, \gamma_1 \rangle, \langle \gamma_2, \gamma_2 \rangle, \langle \gamma_2, \gamma_3 \rangle \| \langle \gamma_3, \gamma_2 \rangle, \langle \gamma_3, \gamma_3 \rangle, \langle \gamma_3, \gamma_4 \rangle, \langle \gamma_3, \gamma_5 \rangle, \langle \gamma_3, \gamma_1 \rangle)$
6	19	$(\langle \gamma_1, \gamma_5 \rangle, \langle \gamma_1, \gamma_6 \rangle, \langle \gamma_1, \gamma_1 \rangle, \langle \gamma_1, \gamma_2 \rangle, \langle \gamma_1, \gamma_3 \rangle, \langle \gamma_1, \gamma_4 \rangle \| \langle \gamma_4, \gamma_1 \rangle, \langle \gamma_4, \gamma_6 \rangle \| \langle \gamma_6, \gamma_4 \rangle, \langle \gamma_6, \gamma_3 \rangle, \langle \gamma_6, \gamma_2 \rangle \| \langle \gamma_2, \gamma_6 \rangle, \langle \gamma_2, \gamma_5 \rangle \| \langle \gamma_5, \gamma_2 \rangle, \langle \gamma_5, \gamma_3 \rangle, \langle \gamma_5, \gamma_4 \rangle, \langle \gamma_5, \gamma_5 \rangle, \langle \gamma_5, \gamma_6 \rangle, \langle \gamma_5, \gamma_1 \rangle)$
6	22	$(\langle \gamma_1, \gamma_5 \rangle, \langle \gamma_1, \gamma_6 \rangle, \langle \gamma_1, \gamma_1 \rangle, \langle \gamma_1, \gamma_2 \rangle, \langle \gamma_1, \gamma_3 \rangle \| \langle \gamma_3, \gamma_1 \rangle, \langle \gamma_3, \gamma_6 \rangle, \langle \gamma_3, \gamma_5 \rangle, \langle \gamma_3, \gamma_4 \rangle, \langle \gamma_3, \gamma_3 \rangle, \langle \gamma_3, \gamma_2 \rangle \| \langle \gamma_2, \gamma_3 \rangle, \langle \gamma_2, \gamma_2 \rangle, \langle \gamma_2, \gamma_1 \rangle, \langle \gamma_2, \gamma_6 \rangle, \langle \gamma_2, \gamma_5 \rangle \| \langle \gamma_5, \gamma_2 \rangle, \langle \gamma_5, \gamma_3 \rangle, \langle \gamma_5, \gamma_4 \rangle, \langle \gamma_5, \gamma_5 \rangle, \langle \gamma_5, \gamma_6 \rangle, \langle \gamma_5, \gamma_1 \rangle)$
6	29	$(\langle \gamma_1, \gamma_3 \rangle, \langle \gamma_1, \gamma_2 \rangle, \langle \gamma_1, \gamma_1 \rangle, \langle \gamma_1, \gamma_6 \rangle, \langle \gamma_1, \gamma_5 \rangle, \langle \gamma_1, \gamma_4 \rangle \| \langle \gamma_4, \gamma_1 \rangle, \langle \gamma_4, \gamma_6 \rangle, \langle \gamma_4, \gamma_5 \rangle, \langle \gamma_4, \gamma_4 \rangle, \langle \gamma_4, \gamma_3 \rangle, \langle \gamma_4, \gamma_2 \rangle \| \langle \gamma_2, \gamma_4 \rangle, \langle \gamma_2, \gamma_3 \rangle, \langle \gamma_2, \gamma_2 \rangle, \langle \gamma_2, \gamma_1 \rangle, \langle \gamma_2, \gamma_6 \rangle, \langle \gamma_2, \gamma_5 \rangle \| \langle \gamma_5, \gamma_2 \rangle, \langle \gamma_5, \gamma_1 \rangle, \langle \gamma_5, \gamma_6 \rangle, \langle \gamma_5, \gamma_5 \rangle, \langle \gamma_5, \gamma_4 \rangle, \langle \gamma_5, \gamma_3 \rangle \| \langle \gamma_3, \gamma_5 \rangle, \langle \gamma_3, \gamma_4 \rangle, \langle \gamma_3, \gamma_3 \rangle, \langle \gamma_3, \gamma_2 \rangle, \langle \gamma_3, \gamma_1 \rangle)$
7	29	$(\langle \gamma_1, \gamma_4 \rangle, \langle \gamma_1, \gamma_5 \rangle \| \langle \gamma_5, \gamma_1 \rangle, \langle \gamma_5, \gamma_7 \rangle, \langle \gamma_5, \gamma_6 \rangle, \langle \gamma_5, \gamma_5 \rangle, \langle \gamma_5, \gamma_4 \rangle, \langle \gamma_5, \gamma_3 \rangle, \langle \gamma_5, \gamma_2 \rangle \langle \gamma_2, \gamma_5 \rangle, \langle \gamma_2, \gamma_6 \rangle, \langle \gamma_2, \gamma_7 \rangle \| \langle \gamma_7, \gamma_2 \rangle, \langle \gamma_7, \gamma_1 \rangle, \langle \gamma_7, \gamma_7 \rangle, \langle \gamma_7, \gamma_6 \rangle, \langle \gamma_7, \gamma_5 \rangle, \langle \gamma_7, \gamma_4 \rangle, \langle \gamma_7, \gamma_3 \rangle \| \langle \gamma_3, \gamma_7 \rangle, \langle \gamma_3, \gamma_6 \rangle, \langle \gamma_3, \gamma_5 \rangle, \langle \gamma_3, \gamma_4 \rangle \| \langle \gamma_4, \gamma_3 \rangle, \| \langle \gamma_4, \gamma_4 \rangle, \langle \gamma_4, \gamma_5 \rangle, \langle \gamma_4, \gamma_6 \rangle, \langle \gamma_4, \gamma_7 \rangle, \langle \gamma_4, \gamma_1 \rangle)$
7	37	$(\langle \gamma_1, \gamma_4 \rangle, \langle \gamma_1, \gamma_3 \rangle, \langle \gamma_1, \gamma_2 \rangle, \langle \gamma_1, \gamma_1 \rangle, \langle \gamma_1, \gamma_7 \rangle, \langle \gamma_1, \gamma_6 \rangle, \langle \gamma_1, \gamma_5 \rangle \| \langle \gamma_5, \gamma_1 \rangle, \langle \gamma_5, \gamma_7 \rangle, \langle \gamma_5, \gamma_6 \rangle, \langle \gamma_5, \gamma_5 \rangle, \langle \gamma_5, \gamma_4 \rangle, \langle \gamma_5, \gamma_3 \rangle, \langle \gamma_5, \gamma_2 \rangle \| \langle \gamma_2, \gamma_5 \rangle, \langle \gamma_2, \gamma_4 \rangle, \langle \gamma_2, \gamma_3 \rangle, \langle \gamma_2, \gamma_2 \rangle, \langle \gamma_2, \gamma_1 \rangle, \langle \gamma_2, \gamma_7 \rangle, \langle \gamma_2, \gamma_6 \rangle, \| \langle \gamma_6, \gamma_2 \rangle, \langle \gamma_6, \gamma_1 \rangle, \langle \gamma_6, \gamma_7 \rangle, \langle \gamma_6, \gamma_6 \rangle, \langle \gamma_6, \gamma_5 \rangle, \langle \gamma_6, \gamma_4 \rangle, \langle \gamma_6, \gamma_3 \rangle \| \langle \gamma_3, \gamma_6 \rangle, \langle \gamma_3, \gamma_5 \rangle, \langle \gamma_3, \gamma_4 \rangle \| \langle \gamma_4, \gamma_3 \rangle, \langle \gamma_4, \gamma_4 \rangle, \langle \gamma_4, \gamma_5 \rangle, \langle \gamma_4, \gamma_6 \rangle, \langle \gamma_4, \gamma_7 \rangle, \langle \gamma_4, \gamma_1 \rangle)$

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Appendix A

When the number of nodes in the factor graph G are between 4 to 7, the problem of finding all cycles of length 7 to $|V(G)|^2$ in an OTIS-G can be found by exploiting the similar methods mentioned already through the proof of Theorem 3. Nevertheless, there are a couple of special cases which cannot be resolved directly by using those methods. Table 1 demonstrates explicitly a detailed description of each case and a direct solution for making the desired cycle at any special case. As said already in Section 3 of this paper, the other cycles can be made in the same way explained in the proof of Theorem 3. (It is worthy to mention that to create a specific cycle, for example a cycle of length 13 within the OTIS-C_4, both cycles of length 3 and 4 in the factor graph G must be exploited, e.g. $C_3 = \langle \gamma_1, \gamma_3, \gamma_2 \rangle$ within the group γ_3.)

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