Analytical Leakage-Aware Thermal Modeling of a Real-Time System

Morteza Mohaqeqi, *Student Member, IEEE*, Mehdi Kargahi, *Member, IEEE*, and Ali Movaghar, *Senior Member, IEEE*

Abstract— We consider a firm real-time system with a single processor working in two power modes depending on whether it is idle or executing a job. The system is equipped with dynamic thermal management through a cooling subsystem which can switch between two cooling modes. Real-time jobs which arrive to the system have stochastic properties and are prone to soft errors. A successful job is one that enters the system and completes its execution with no timing or soft error. Appropriateness of the system is evaluated based on its performance, temperature behavior, reliability, and energy consumption. It is noteworthy that these criteria have mutual interactions to each other: the stochastic nature of the system affects the success ratio of jobs beside the system dynamic power, the leakage as well as dynamic power impacts the processor temperature, this temperature affects the leakage power, the cooling subsystem power, and the soft error rate, which the latter in turn impacts the system reliability and the success ratio of jobs. This paper proposes an analytical evaluation method with a Markovian view to the system which considers these reciprocal effects. A number of simulation experiments are carried out to validate the accuracy of the proposed method.

Index Terms—Analytical modeling, Firm real-time system, Leakage power, Performance, Temperature.

—————————— ◆ ——————————

1 INTRODUCTION

SCALING down of the VLSI technology to the deep sub-micron domain has led to higher chip power density and leakage power. In recent years, with a 30% improvement in the processor frequency, power density has been approximately doubled [1], resulting in increased temperature. The advances beside the chip temperature have significant impacts on the measures of performance [2], reliability [3] and power consumption [4] of computing systems.

Awareness to the amount of such temperature-related measures are especially much more important in real-time embedded systems working in dynamic environments with uncertain properties. Examples are systems running critical applications in the space and defense domains as well as less critical multimedia applications. The uncertainties of these systems can be observed in the arrival pattern of jobs, job characteristics (such as execution times and deadlines), and environmental transient faults, which are usually described by stochastic models [5], [6], [7]. One important issue in such real-time systems is to find the effects of the

stochastic properties on the aforementioned temperature-related measures, as the temperature seriously affects the system behavior.

On one hand, temperature has a considerable effect on the leakage current in MOS transistors. An example in [8] shows that, when the ambient temperature is 35℃, an increase of 10℃ can cause 126% rise in the leakage current. Accordingly, increased power density, not only directly raises the energy consumption, but also it has an indirect impact on the energy through increased temperature. This interdependency, if not appropriately estimated and taken into account, may result a positive feedback loop which can cause thermal runaway.

On the other hand, chip temperature affects both the rates of permanent failures and transient faults [1]. Rosing et al. [3] show such an effect on the hard error rate in SoCs. Also, similar effect on the soft error rate (SER) has been investigated in [9], [10]. For example, Chandra and Aitken [9] report the sensitivity of SER to the temperature in different voltage levels.

The above concerns have been widely studied in the context of real-time systems emphasizing the system performance. For example, in [11], [12], and [13], respectively, the energy consumption, temperature runaway, and error rate affecting the system reliability have been considered. Most of these studies use speed and voltage scaling (through dynamic voltage scaling (DVS) [14] or adaptive body biasing (ABB) [15]) beside appropriate scheduling algorithms [16], [17] to enhance the system behavior. However, to better employ these techniques, some integrated models are needed to sketch the interdependency between the system specification (such as the processor voltage and speed, scheduling algorithm, workload parameters, etc.), and the measures

————————————————

- M. Mohaqeqi is with the DRTS Lab, School of Electrical and Computer Engineering, College of Engineering, University of Tehran, P.O. Box 14399-57131, Tehran, Iran. E-mail: m.mohaqeqi@ut.ac.ir.
- M. Kargahi is with the DRTS Lab, School of Electrical and Computer Engineering, College of Engineering, University of Tehran, P.O. Box 14399-57131, Tehran, Iran. E-mail: kargahi@ut.ac.ir. His second affiliation is School of Computer Science, Institute for research in fundamental sciences (IPM), P.O. Box 19395-5746, Tehran, Iran.
- A. Movaghar is with the Department of Computer Engineering, Sharif University of Technology, P.O. Box 11365-9517, Tehran, Iran. E-mail: movaghar@sharif.edu.

of system behavior (such as deadline violation ratio, reliability, and energy consumption). Even without consideration of the temperature effects, investigation of such interdependency in an analytical manner is not straightforward [5], [18], [19].

In this paper, we consider the analytical study of a stochastic single-processor firm real-time (FRT) system with non-preemptive independent jobs, i.e., systems in which the jobs missing their deadlines are thrown away immediately. We consider the class of non-idling service-time independent scheduling algorithms that, when the system capacity is not full, do not decide on accepting or rejecting a job on its arrival. In particular, we concentrate on the first-come first-served (FCFS) scheduling algorithm. The stochastic behavior of the system is seen in the workload characteristics as well as soft errors. The system processor has the DVS capability which can be used for putting the system in a low-power mode when the processor becomes idle. Also, the system is equipped with a dynamic thermal management (DTM) mechanism using a cooling subsystem. Both DVS and DTM help to overcome the excessive energy usage, temperature rise up, and reliability loss. The main purpose of this study is to propose an analytical method to identify the effects of stochastic characteristics of the FRT system on its performance and temperature behavior, and in turn, on the system reliability and energy consumption.

In summary, the main contributions of this paper are as follows: rather than proposing a new algorithm for improving the system energy efficiency and controlling its temperature and reliability, we provide an analytical method for evaluating the system behavior, and finding the appropriate configurations to reach the goals. The analytical method takes into account the effects of real-time workload, processor dynamic power, leakage power and temperature, as well as DTM (through a cooling subsystem), altogether, in a Markovian view to the FRT system. Some important probabilistic performance-, temperature-, reliability-, and energy-related measures of the system are calculated. The proposed analytical method is quite accurate for specific patterns of job deadlines and highly accurate approximate for the other patterns. The measures are studied for different system parameters and the validity of results as well as the accuracy-level of approximations is shown through extensive system-level and microarchitecture-level simulations. To the best of the authors' knowledge, the current work is the first which, based on a Markovian view, studies the stochastic behavior of a FRT system considering the temperature effects.

The rest of this paper is organized as follows. Next section reviews the related literature. Section 3 provides our system model and the favorite performance-, temperature-, reliability-, and energy-related measures. Section 4 describes the timing behavior of the system which provides required means for thermal analysis. In Section 5, we propose our stochastic thermal analysis method. The experimental evaluation of the method is provided in Section 6. Finally, the paper concludes in Section 7.

2 RELATED WORK

In this section, we first review some related literature on the modeling and management of energy consumption, performance, and reliability with an emphasis on real-time systems. Afterwards, we focus on the temperature-related works, including a review on the studies that consider the effects of chip temperature on the system power consumption, reliability and performance of non real-time as well as real-time systems, from both deterministic and stochastic points of view.

The interaction between performance and energy efficiency of deterministic [17], [20], [21] and stochastic [5], [22], [23] computing systems has been widely studied. In the context of systems with stochastic properties, which is the focus of this paper attention, an optimization-based approach is proposed in [23] for balancing the mean response time and mean energy consumption in a stochastic non real-time system. In [22], an analytical DVS-based stochastic dynamic power management technique is devised for making a tradeoff between the mean energy consumption and performance of a real-time system. Also, in [5], a performance optimization method based on an analytical approach is proposed for utility-based real-time systems. In both [22] and [5], the processor energy consumption is population-dependent. The interaction between reliability and energy efficiency of real-time systems has also been investigated [13], [24].

Considering the temperature issues, several studies have been performed on the power consumption [4], [25], [26], reliability [3], [9], [27], [28] and performance [29], [30] of non real-time and real-time systems.

In the context of joint thermal and power management, it has been shown that leakage power is highly dependent on temperature [4]. Therefore, recent studies have utilized the temperature model to calculate leakage power [4] and accordingly address the problem of temperature-aware power management [26]. Liao, He and Lepak in [4] investigate the dependency of leakage power on processor temperature and propose a formula that describes this dependency as a nonlinear function. Later, Quan and Zhang [2] estimated the same measure with a linear function and mixed this estimated function with the lumped RC model [31] in order to have a simpler leakage-aware temperature analysis. To minimize the peak temperature through reducing the power consumption, some works (e.g. [25]) have employed DVS-based scheduling. Also, in real-time systems, Huang and Quan [26] propose a scheduling algorithm that aims at energy minimization with proper frequency selection for each task with regard to a temperature constraint.

Besides, some authors have studied the impact of temperature on the system reliability. Rosing, Mihic and De Micheli [3] have studied some temperature-dependent failure mechanisms, including electro-migration, time-dependent dielectric breakdown and thermal cycling in order to make a tradeoff between the reliability and power consumption of a SoC. With focus on thermal cycling failure mechanism, authors in [28] present a temperature-aware reliability optimization based on a

steady-state temperature analysis for applications with periodic power profile. Chip temperature can also affect the SER [9], [27], [32], which in turn, can impact the chip reliability. However, a few studies have focused on the SER dependence on temperature. Chandra and Aitken [9] studied the impact of technology scaling on the soft error susceptibility and reported increased SER when temperature increases. In addition, the effect of temperature on soft errors has been studied in [27], where it is shown that, although increased temperature leads to SER rising in general, the dependency is device-specific and may vary from one device of a vendor to the other.

The impact of temperature constraints on the system performance has been investigated by several recent researches. The methods guarantee that the temperature does not exceed a determined threshold. In [33], a frequency assignment method is proposed for MPSoCs in order to maximize the system performance regarding a temperature constraint. In [29], the feasibility of a speed schedule for a set of real-time tasks in terms of some temperature constraints has been investigated. Also, in [2], some feasibility analysis, including necessary and sufficient conditions for the schedulability test, has been proposed for an arbitrary periodic task set with hard time constraints.

While many recent works have studied the thermal-aware performance and energy modeling [3], [4], [31] and management [11], [25], [26], [30], [33] in a deterministic manner, few studies [34], [35], [36], [37] have considered thermal aspects in stochastic systems. Zhang and Chatha [35] aim to minimize the makespan of a task set with uncertain execution times, subject to guaranteeing an upper bound on the probability of violating a specified critical temperature. They consider a fixed set of jobs with equal release times and provide an offline solution for selecting a voltage/frequency for each job. Also, in [36], a fixed set of non real-time jobs with nondeterministic execution times is considered under temperature constraints. To minimize the peak temperature, the approach divides the jobs into cool and hot subsets and performs job sequencing based on average job execution times. Jung, Rong and Pedram [34] propose a stochastic model for a thermally managed non real-time multicore system. In their model, the thermal behavior of the system is captured by some discrete states, each one representing a range of temperatures.

In the context of stochastic real-time systems, a model of mixed real-time periodic and non real-time aperiodic tasks under a tight thermal constraint has been investigated [37]. The proposed approach initially obtains a safe speed under which the temperature constraint is guaranteed to be preserved. Then, with the previously obtained speed, a queuing model for the non real-time aperiodic tasks is solved and the response time is assessed. This way, the work separates the analysis of the thermal behavior of the system from its performance analysis.

The main differences between the current work and the aforementioned studies on stochastic thermal analysis can be summarized as follows: This study builds its temperature analysis on the basis of a queuing model for a DTM-enabled FRT system with stochastic properties, it provides a complete description of the thermal behavior of the system through analytical calculation of the respective temperature probability density function, it considers the possible impacts of the stochastic real-time workload, leakage and dynamic power consumption, processor temperature, cooling subsystem, and reliability of the real-time system altogether, beside providing a rich set of respective measures.

3 SYSTEM MODEL AND MEASURES

This section introduces the system model, including the models of job and service, processor power consumption and temperature, cooling subsystem, and soft errors. Then, our favorite performance-, temperature-, reliability-, and energy-related measures are formulated.

Throughout this paper, we assume statistical equilibrium and use $n \in \mathcal{N}$, $\tau \in \mathcal{R}$ and $t \in \mathcal{R}^+$, where $\mathcal{N}$, $\mathcal{R}$ and $\mathcal{R}^+$ are the sets of natural numbers, real numbers, and nonnegative real numbers, respectively. Also, we use the terms speed and service rate interchangeably.

3.1 System Model
We consider a single-processor queue with an arbitrary capacity K ($K < \infty$).

Job and service model: Jobs of this system are defined as $J = (a, c, \theta)$, where a, c, and θ, are the job arrival time, computation (execution) time, and relative deadline, respectively. A state of the system is shown with n, where $n \leq K$ is the number of jobs in the system. More precise definitions of these parameters are presented in the following.

The job arrival times (a) follow a state-dependent Poisson process with rate λ_n. An arriving job which finds the system full (i.e., finds $n = K$) is blocked and must leave the system immediately. Jobs entering the system are served in the order of their arrival, i.e., the service discipline in the system is FCFS.

Each job has an exponential execution-time (c) with probability density function (PDF) $f_c(t)$ and an expected value $\bar{c}$. Throughout this paper, we normalize all timing parameters with respect to $\bar{c}$.

Further, each job has a relative deadline θ which is the difference between its absolute deadline (named deadline for short) and arrival time. θ is a random variable with a cumulative distribution function (CDF) $G(t)$ and average $\bar{\theta}$. We assume that $G(t)$ is a general CDF which may have a mass at the infinity, namely, in general, $P(\theta = \infty) = 1 - \lim_{t \to \infty} G(t) \geq 0$. A job is immediately thrown away of the system as soon as it misses its deadline irrespective of whether or not it is being served. In other words, each job has a deadline until the end of its service. Job execution times and relative deadlines form sequences of independently and identically distributed (i.i.d.) random variables which are mutually independent.

Processor and cooling subsystem models: The processor has a discrete set of speeds $M = \{\mu_1, \dots, \mu_l\}$, where l is the number of speed levels. Also, two modes of

operation are identified:

1. *Idle mode*: when there is no job in the system and the processor is idle at speed level $\mu_I \in$ M; and
2. *Active mode*: when the processor executes some jobs in the system at speed $\mu_A \in$ M.

The processor speed is characterized by the respective voltage level from the set $\Upsilon = \{v_1, \ldots, v_l\}$. We show the supply voltages of idle and active modes by $v_I \in \Upsilon$ and $v_A \in \Upsilon$, respectively. It is supposed that $v_I \leq v_A$. (We ignore the possible overheads of switching between v_I and v_A.)

The processor has a predefined power function $Pow(v_k)$, $v_k \in \Upsilon$, which can be considered as the energy consumption of the processor at speed level $\mu_k \in$ M and a fixed (either idle or active) mode, normalized with respect to the time unit. This processor power function consists of two parts, dynamic power ($Pow_{dyn}(v_k)$) and leakage power ($Pow_{leak}(v_k)$), i.e., we have:

$$Pow(v_k) = Pow_{dyn}(v_k) + Pow_{leak}(v_k), \qquad (1)$$

where both dynamic and leakage power functions depend on the processor supply voltage. This relation for the former function is formulated as [38]:

$$Pow_{dyn}(v_k) = \begin{cases} C_0 v_k^3, & \text{if the processor is in idle mode} \\ C_0' v_k^3, & \text{if the processor is in active mode} \end{cases}, \qquad (2)$$

where C_0 and C_0' are processor-specific constants and $v_k \in \Upsilon$ is the kth voltage level of the processor. On the other hand, the latter function is estimated as [4]:

$$Pow_{leak}(v_k) = N_{gate} I_{leak} v_k, \qquad (3)$$

where N_{gate} is the number of gates in the processor and I_{leak} is the leakage current. Let define the random variable T as

$T \equiv$ the steady state temperature of the processor. (4)

Then, I_{leak} is calculated by the following formula:

$$I_{leak} = I_s \left(\beta_1 T^2 e^{\frac{\alpha_1 v_k + \alpha_2}{T}} + \beta_2 e^{\alpha_3 v_k + \alpha_4} \right), \qquad (5)$$

where I_s is the leakage current at a certain reference point and β_1, β_2, α_1, α_2, α_3 and α_4 are empirically determined processor-specific constants.

Based on (3) and (5), there is a nonlinear relationship between the leakage power and temperature. However, it has been shown in [2] that the following linear approximation of (3) introduces negligible error into the calculation of the processor leakage power:

$$Pow_{leak}(v_k) = (C_1(k) + C_2(k)T)v_k, \qquad (6)$$

where $C_1(k)$ and $C_2(k)$ are some processor-specific voltage-sensitive constants. Thus, leakage power is a function of both supply voltage and processor temperature. Then, at a constant processor temperature T, the instantaneous processor power consumption at voltage v_k is obtained using (2), (6), and (1). However, the processor temperature may vary over time.

Because of reliability risks and energy wastage due to increased temperature, some types of DTM might be required. In [46], a cooling subsystem has been employed for reducing the processor leakage power through DTM. We also consider a cooling subsystem with a discrete set of cooling modes CM $= \{m_1, \ldots, m_r\}$, where r is number of the modes. The subsystem switches between two cooling modes m_L (low power) and m_H (high power) belonging to CM, based on the processor temperature with respect to a

temperature threshold T_{th}. (This idea can be extended to switching among arbitrary number of cooling modes based on multiple temperature thresholds with minor algebraic efforts with respect to what follows.) The cooling subsystem has a predefined power function $CPow(m_i)$, $m_i \in$ CM. Normally, the subsystem resides in m_L, however, when the processor is in the active mode, in the case that the processor temperature exceeds T_{th}, the cooling mode switches to m_H. This switching causes a slower temperature rise up, and consequently, a lower steady-state temperature comparing to the case that we ignore the switching. The cooling mode remains the same until the system goes to the idle mode that the cooling subsystem switches back to m_L. As we will see analytically in the following, m_L, m_H, and T_{th} are design parameters which affect the system power consumption, temperature, and reliability.

We continue with the temperature behavior of the system. Suppose that the processor works at some constant voltage v_k, a fixed (either idle or active) mode, and a fixed cooling mode (either m_L or m_H) for a duration, and t is some time in that duration. We adopt the well-known lumped RC model [31] to describe the transient behavior of the processor temperature as:

$$\frac{dT(t)}{dt} = \rho Pow(t) - \beta(m_i)T(t), \qquad (7)$$

where ρ is a processor-specific thermal coefficient, $\beta(m_i)$ is a processor- and cooling mode-specific thermal coefficient, and $Pow(t)$ and $T(t)$ are the instantaneous power consumption and temperature of the processor at the mentioned voltage level. Without loss of generality and similar to [2], in (7), we assume that the temperature has been scaled such that the ambient temperature is considered zero. (Otherwise, an offset equal to the ambient temperature is entered in (7) and the subsequent related formulations.) Using (2), (6), (1), and (7), we obtain:

$$\frac{dT(t)}{dt} = \rho\big(Pow_{dyn}(v_k) + C_1(k)v_k\big)$$
$$- (\beta(m_i) - \rho C_2(k)v_k)T(t)$$
$$= M(v_k) - N(v_k, m_i)T(t). \qquad (8)$$

Suppose the temperature at an initial time t_0 is $T(t_0)$. Also, consider that during time interval $[t_0, t]$, the processor remains in the same mode, it remains in the same voltage level v_k, and the cooling mode m_i is fixed. Then the temperature at time t can be obtained through solving differential equation (8) as follows:

$$T(t) = \frac{M(v_k)}{N(v_k, m_i)} + \left(T(t_0) - \frac{M(v_k)}{N(v_k, m_i)}\right)e^{-N(v_k, m_i)(t - t_0)}. \qquad (9)$$

As we consider *i*) two modes of operations, and possibly two levels of voltages for the processor (v_I for the idle mode and v_A for the active mode), and *ii*) two cooling modes (m_L and m_H), different parameters in the temperature formulations might be used depending on the possible mode combinations. Thus, with some simplifications in the notations, the transient temperature for the processor in the idle mode can be written as:

$$T_I(t) = T_{Min} + (T(t_0) - T_{Min})e^{-N(v_I, m_L)(t - t_0)}, \qquad (10)$$

Moreover, if the processor works in the active mode, the transient temperature is calculated as:

$$T_A(t) = \begin{cases} T_{Max_L} + \left(T(t_0) - T_{Max_L}\right)e^{-N(v_A, m_L)(t-t_0)}, \\ \qquad \text{if } T(t_0) < T_{th} \text{ and } t - t_0 \leq \omega \\ T_{Max_H} + \left(T_{th} - T_{Max_H}\right)e^{-N(v_A, m_H)(t-t_0-\omega)}, \\ \qquad \text{if } T(t_0) < T_{th} \text{ and } t - t_0 > \omega \\ T_{Max_H} + \left(T(t_0) - T_{Max_H}\right)e^{-N(v_A, m_H)(t-t_0)}, \quad \text{if } T(t_0) > T_{th} \end{cases} \quad (11)$$

where

$$\omega = -\frac{1}{N(v_A, m_L)} \ln \frac{T_{Max_L} - T_{th}}{T_{Max_L} - T(t_0)}, \quad (12)$$

is the time required to reach from $T(t_0)$ to T_{th}. Note that $T_{Min} = M(v_I)/N(v_I, m_L)$, $T_{Max_L} = M(v_A)/N(v_A, m_L)$, and $T_{Max_H} = M(v_A)/N(v_A, m_H)$, respectively, are the temperatures to which the processor will reach if it sufficiently resides in the idle mode, in the active mode with cooling mode m_L, and in the active mode with cooling mode m_H ($T_{Min} \leq T_{Max_H} \leq T_{Max_L}$). In other words, the processor steady-state temperature varies between T_{Min} and T_{Max_H}.

Error model: We consider the impact of soft errors on the jobs. It has been shown in [9] that the temperature affects the SER through Qcrit, i.e. the minimum charge that is needed to flip one bit. The radiation particles are as major causes of such bit flips. Since radiation particle hit follows a Poisson process [39], the soft error occurrence is also commonly modeled by a Poisson distribution [7], [39]. Suppose the PDF of soft error inter-occurrence time at temperature $T(t)$ as $\epsilon(t) = \varrho(T(t))e^{-\varrho(T(t))t}$, where $\varrho(T(t))$ is the SER at temperature $T(t)$.

We consider *hidden* fault model [40], where the soft errors of affected jobs remain undetected until the job leaves the system. However, even if the affected job has met its deadline, its erroneous result will not be considered by the system; rather the result is discarded. Define $p_{se}(T(t_0), c)$ as the probability that a job which starts its service at temperature $T(t_0)$ and has execution-time c (i.e. completes at time $t_0 + c$) is affected by at least one soft error. Obviously the system is in the active mode when executing the job. Thus, the probability that the job suffers from soft error is calculated as:

$$p_{se}(T(t_0), c) = \int_0^c \epsilon(t) \, dt = \int_0^c \varrho\left(T_A(t_0+t)\right)e^{-\varrho(T_A(t_0+t))t} \, dt, (13)$$

where $T_A(.)$ is defined in (11).

3.2 Measures

This subsection introduces the system performance and temperature-related measures, beside their relation to the system energy and reliability. We begin with the definition of a basic performance variable. Let

$V \equiv$ the time an arriving job with infinite (no) deadline, in the long run, must wait before it completes its service.(14)

V is called the job offered sojourn (response) time. We assume $V = \infty$ if the arriving job is blocked due to full system. We will be interested in finding the CDF of V

$$F_V(t) = P(V \leq t), \quad (15)$$

or equivalently, its PDF $f_V(t)$.

More specific measures of performance may also be defined using $f_V(t)$. In particular, we will be interested in the probability of missing deadline, defined as

$$\alpha_d = P(\theta < V < \infty) = \int_0^\infty f_V(t)G(t) \, dt . \quad (16)$$

α_d represents the steady-state probability that a job misses its deadline. Another important measure of

performance is the probability of blocking α_b, defined as

$$\alpha_b = P(V = \infty) = 1 - \lim_{t \to \infty} F_V(t). \quad (17)$$

α_b is interpreted as the steady-state probability that an arriving job is rejected due to full system.

The above performance measures assume no significance for the impact of processor temperature on the success ratio of jobs which meet their deadline. Rather, as indicated before, the temperature affects the SER, and thus, the system reliability. In the following, we first discuss the steady-state thermal behavior of the system, and then, investigate its effect on the reliability.

As defined in (4), T is the random variable which defines the steady-state temperature of the processor. We will be interested in finding the CDF of T

$$H_T(\tau) = P(T \leq \tau), \quad (18)$$

or equivalently, the PDF

$$h_T(\tau) = \frac{dH_T(\tau)}{d\tau}. \quad (19)$$

As indicated before, the processor works either in the idle or in the active mode. Let define the following probabilities:

$\pi_I \equiv$ the steady state probability that the processor is in the idle mode,

$\pi_A \equiv$ the steady state probability that the processor is in the active mode . (20)

Also, we define the following random variables:

$T_I \equiv$ the long run temperature of the processor when it is in the idle mode,

$T_A \equiv$ the long run temperature of the processor when it is in the active mode. (21)

Assuming $h_{T_I}(\tau)$ and $h_{T_A}(\tau)$, respectively, as the PDFs of T_I and T_A, we can write the PDF of the processor temperature defined in (19) as:

$$h_T(\tau) = \pi_I h_{T_I}(\tau) + \pi_A h_{T_A}(\tau). \quad (22)$$

We will discuss the way of calculating $h_{T_I}(\tau)$ and $h_{T_A}(\tau)$ in more details in the following sections.

Now, we aim to investigate the aforementioned effect of processor temperature on the SER, and therefore, on the system reliability. Consider the following definition:

$W \equiv$ the time an arriving job with infinite (no) deadline, in the long run, must wait before it starts its service, (23)

with $f_W(t)$ as its PDF. Since we consider hidden fault model, namely the occurred faults are not detected until the completion of the job, we are only interested in the jobs which meet their deadline. The probability that, when the system is in the idle mode, an arriving job to the system meets its deadline and is affected by soft errors can be calculated as:

$$P_{SE}(V \leq \theta | n = 0) = \int_{T_{Min}}^{T_{Max_H}} h_{T_I}(\tau) \int_0^\infty f_W(t, n = 0)$$
$$\int_0^\infty f_c(u) \left(1 - G(t+u)\right) p_{se}(\tau, u) \, du \, dt \, d\tau, \quad (24)$$

where n is the state of the system as defined in Section 3.1, $f_c(u) = \mu_A e^{-\mu_A u}$ is the PDF of job execution-time, $G(x)$ is the CDF of job relative deadline with a PDF $g(t)$, τ is the temperature of the processor when the job starts its service, which, as defined in the processor model above, $\tau \in [T_{Min}, T_{Max_H}]$ in the steady-state, and $p_{se}(.)$ is defined in

(13). Further, $f_W(t, n = 0)$ is defined as $f_W(t|n = 0)$ multiplied by the probability that an incoming job finds the system in the idle mode (see Lemma 2 in Section 4.2). Similarly, the probability that, when the system is in the active mode, an arriving job to the system meets its deadline and is affected by soft errors is obtained as:

$$P_{SE}(V \le \theta | 0 < n < K) = \int_{T_{Min}}^{T_{Max_H}} h_{T_A}(\tau) \int_0^\infty f_W(t, 0 < n < K)$$

$$\int_0^\infty f_c(u)\left(1 - G(t+u)\right)p_{se}(\tau, u)\, du\, dt\, d\tau. \quad (25)$$

Again, $f_W(t, 0 < n < K)$ is defined as $f_W(t|0 < n < K)$ multiplied by the probability that an incoming job finds the system in the active mode (see Lemma 2). Then, using (24) and (25), the fraction of jobs completed in a timely manner but affected by soft errors, namely α_{se} is calculated as follows:

$$\alpha_{se} = P_{SE}(V \le \theta | n = 0) + P_{SE}(V \le \theta | 0 < n < K). \quad (26)$$

Accordingly, the total probability of loss, considering both timing and soft errors is obtained as:

$$\alpha = \alpha_d + \alpha_b + \alpha_{se}. \quad (27)$$

We also define the following energy-related measures which may be affected by the processor temperature. Based on (1) and the cooling subsystem model, the average power consumption of the system is calculated as:

$$\overline{Pow} = \overline{Pow_{dyn}} + \overline{Pow_{leak}} + \overline{Pow_{cool}}, \quad (28)$$

where, using (2) and (20), we have

$$\overline{Pow_{dyn}} = \pi_I Pow_{dyn}(v_I) + \pi_A Pow_{dyn}(v_A), \quad (29)$$

using (6), (20), and (21), we can write

$$\overline{Pow_{leak}} = \pi_I \int_{T_{Min}}^{T_{Max_H}} h_{T_I}(\tau) Pow_{leak}(v_I)\, d\tau$$

$$+ \pi_A \int_{T_{Min}}^{T_{Max_H}} h_{T_A}(\tau) Pow_{leak}(v_A)\, d\tau, \quad (30)$$

and $\overline{Pow_{cool}}$ is the average cooling power defined below. Using (20) and (21), we can calculate the probability that the cooling subsystem is in the high power mode as:

$$\mathcal{P}(T_{th}) = P(T_A \ge T_{th}) = \pi_A \int_{T_{th}}^{T_{Max_H}} h_{T_A}(\tau) d\tau. \quad (31)$$

Therefore, $\overline{Pow_{cool}}$ can be calculated as:

$$\overline{Pow_{cool}} = CPow(m_L)\left(1 - \mathcal{P}(T_{th})\right) + CPow(m_H)\mathcal{P}(T_{th}). \quad (32)$$

We also define Δ as the ratio between the energy consumption rate and the successful job rate as:

$$\Delta = \frac{\overline{Pow}}{(1-\alpha)\overline{\lambda}}, \quad (33)$$

where $\overline{\lambda}$ is the average job arrival rate (calculated in Section 4.2) and $(1 - \alpha)$ is the probability of successful completion of a job, i.e. with no timing/soft error (α is defined in (27)). In other words, Δ is the energy spent per successful job.

4 QUEUING MODEL

In the following, first, the formulations of some important conditional performance variables of the FRT system are presented based on the analysis proposed in [5] and [18]. Then, a solution for the queuing model of the system is provided beside the way of calculating some elementary performance measures. One important measure that we extract is the sojourn-time of the system in the idle and active modes, which are used in our temperature analysis in Section 5.

4.1 Conditional Performance Variables

For $t, \varepsilon \in \mathcal{R}^+$ and $n \in \mathcal{N}$, let

$\Psi_n(t, \varepsilon) \equiv$ the probability that one of the jobs in the system misses its deadline during $[t, t + \varepsilon]$, given there are n jobs in the system at time t. (34)

Define

$$\gamma_n(t) = \lim_{\varepsilon \to 0} \frac{\Psi_n(t, \varepsilon)}{\varepsilon}, \quad (35)$$

$$\gamma_n = \lim_{t \to \infty} \gamma_n(t), \quad (36)$$

where γ_n is the (steady-state) rate of missing deadlines, given there are n jobs in the system (including the one being served).

Barrer [19] first derived an expression for γ_n for a system with an unbounded capacity, a Poisson arrival process, a deterministic job relative deadline, and FCFS scheduling algorithm. Barrer's result is extended in [18] to a larger class of systems, namely those with an arbitrary capacity, a state-dependent Poisson arrival process, and a general job relative deadline. The concept of this parameter has also been used in the analysis of some general class of real-time systems in [5] as well as for approximating the performance of systems with other scheduling algorithms in [6] and [41].

Define

$V_n \equiv$ the time an arriving job with infinite (no) deadline, in the long run, must wait before it completes its service, given it finds n jobs in the system. (37)

V_n is called the job offered sojourn-time, given there are n jobs in the system. Let

$$F_{V_n}(t) = P(V_n \le t), \quad (38)$$

$$f_{V_n}(t) = \frac{dF_{V_n}(t)}{dt}. \quad (39)$$

In [18], Movaghar has derived a closed-form solution for the PDF defined in (39). Suppose $\Phi_n(s)$ to be the Laplace transform of $\left[\int_0^t (1 - G(x))dx\right]^n$, i.e.,

$$\Phi_n(s) = \int_0^\infty \left[\int_0^t (1 - G(x))dx\right]^n e^{-st} dt. \quad (40)$$

As shown in [18], $f_{V_n}(t)$ can be calculated as:

$$f_{V_n}(t) = \frac{1}{\Phi_n(\mu_A)}\left[\int_0^t (1 - G(x))dx\right]^n e^{-\mu_A t}. \quad (41)$$

Consider

$W_n \equiv$ the time an arriving job with infinite (no) deadline, in the long run, must wait before it starts its service, given it finds n jobs in the system. (42)

W_n is the job offered waiting-time, given there are n jobs in the system. Let

$$F_{W_n}(t) = P(W_n \le t), \quad (43)$$

$$f_{W_n}(t) = \frac{dF_{W_n}(t)}{dt}. \quad (44)$$

We have the following lemma:

Lemma 1. *The PDF of W_n can be calculated as*:

$$f_{W_n}(t) = \frac{n e^{-\mu_A t}}{\mu_A \Phi_n(\mu_A)}\left[\int_0^t (1 - G(x))dx\right]^{n-1}(1 - G(t)). \quad (45)$$

Proof: According to the definitions of V_n in (37) and W_n in (42), their respective PDFs $f_{V_n}(t)$ in (39) and $f_{W_n}(t)$ in (44), beside the PDF of job execution-time $f_c(t)$ given in
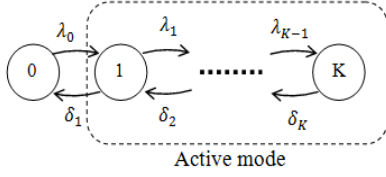
Fig. 1. The FRT system transition-rate graph.

Section 3.1, we have:

$$f_{V_n}(t) = \int_0^t f_{W_n}(u) f_c(t-u) du = (f_{W_n} * f_c)(t), \quad (46)$$

where * is the convolution operator. Using the Laplace transforms, we have:

$$\mathcal{F}_{V_n}(s) = \mathcal{F}_{W_n}(s)\mathcal{F}_c(s), \quad (47)$$

where $\mathcal{F}_c(s) = \frac{\mu_A}{s+\mu_A}$. Therefore, $f_{W_n}(t)$ is obtained as:

$$f_{W_n}(t) = f_{V_n}(t) + f'_{V_n}(t)/\mu_A, \quad (48)$$

where $f_{V_n}(t)$ is given in (41) and $f'_{V_n}(t)$ is the respective derivative. Then, using some algebra, (45) is obtained. □
As will be discussed in the following subsection, $f_{W_n}(t)$ is used in the calculation of α_{se} which is defined in (26).

A closed-form solution to the loss rate γ_n, as defined in (36), can also be obtained as follows [18]:

$$\gamma_n = \begin{cases} 0, & n = 0, \\ n\frac{\Phi_{n-1}(\mu_A)}{\Phi_n(\mu_A)} - \mu_A, & n > 0, \end{cases} \quad (49)$$

where $\Phi_n(.)$ is defined in (40).

4.2 Model Solution and Performance Measures

Long-run behavior of the system introduced in Section 3 can be viewed as a Markovian model which is solved analytically using some standard solution techniques. According to the system model which considers K as the system capacity, we have a Markovian model with $K + 1$ states (see Fig. 1) where $\delta_n = \mu_A + \gamma_n$. Define

$$\pi_n(t) \equiv \text{The probability that there are } n \text{ jobs in the system at time } t. \quad (50)$$

Then, we can write

$$\frac{d\pi_0(t)}{dt} = -\lambda_0\pi_0(t) + (\mu_A + \gamma_1(t))\pi_1(t),$$

$$\frac{d\pi_n(t)}{dt} = \lambda_{n-1}\pi_{n-1}(t) - (\lambda_n + \mu_A + \gamma_n(t))\pi_n(t) \quad (51)$$
$$+(\mu_A + \gamma_{n+1}(t))\pi_{n+1}(t), \quad \text{if } 0 < n < K.$$

Let

$$\pi_n = \lim_{t\to\infty} \pi_n(t). \quad (52)$$

Then, in equilibrium, (51) can be rewritten as follows:

$$0 = -\lambda_0\pi_0 + \delta_1\pi_1,$$
$$0 = \lambda_{n-1}\pi_{n-1} - (\lambda_n + \delta_n)\pi_n + \delta_{n+1}\pi_{n+1}, \text{ if } 0 < n < K. \quad (53)$$

Solving the equilibrium, we obtain:

$$\pi_n = \prod_{i=0}^{n-1} \frac{\lambda_i}{\delta_{i+1}} \pi_0, \text{ if } 0 < n \leq K. \quad (54)$$

π_n is the steady-state probability that there are n jobs in the system. From (54) and using the normalizing condition ($\sum_{n=0}^K \pi_n = 1$), we obtain:

$$\pi_0 = \left(1 + \sum_{n=1}^K \prod_{i=0}^{n-1} \frac{\lambda_i}{\delta_{i+1}}\right)^{-1}. \quad (55)$$

Define the average arrival rate of jobs to the system, which is used in the calculation of Δ in (33), as:

$$\bar{\lambda} = \sum_{i=0}^K \lambda_i \pi_i. \quad (56)$$

Then, the steady-state probability that an incoming job finds n jobs in the system, namely r_n is obtained as:

$$r_n = \frac{\lambda_n\pi_n}{\bar{\lambda}}, \text{ for } 0 \leq n \leq K. \quad (57)$$

Given $f_{V_n}(t)$ as in (39), the PDF of job offered sojourn-time V defined in (14) can then be obtained as:

$$f_V(t) = \sum_{n=0}^{K-1} r_n f_{V_n}(t). \quad (58)$$

Having $f_V(t)$ and using (16), the probability of missing deadline can simply be calculated. Meanwhile, given γ_n as in (49), the probability can also be derived as

$$\alpha_d = \frac{\sum_{n=1}^K \pi_n\gamma_n}{\bar{\lambda}}, \quad (59)$$

which is the average deadline miss rate divided by the average job arrival rate. The probability that an incoming job finds that the system is full, i.e., the blocking probability defined in (17) is simply

$$\alpha_b = r_K. \quad (60)$$

Now, we can proceed with the calculation of α_{se} as defined in (26). We use the following lemma:

Lemma 2. *We can write* $f_W(t, n = 0)$ *and* $f_W(t, 0 < n < K)$ *as follows:*

$$f_W(t, n = 0) = r_0 f_{W_0}(t) = r_0\delta(t), \quad (61)$$

$$f_W(t, 0 < n < K) = \sum_{n=1}^{K-1} r_n f_{W_n}(t), \quad (62)$$

where $\delta(t)$ *is a Dirac delta (impulse) function.*

Proof: Based on the definitions provided in (23), (42), and (57), the proof is immediate. □
Also, the steady-state probability that the system is in the idle and active modes (defined in (20)) can easily be obtained as:

$$\pi_I = \pi_0, \quad (63)$$
$$\pi_A = \sum_{n=1}^K \pi_n. \quad (64)$$

Let

$$S_n \equiv \text{the long run sojourn time of the system in state } n. \quad (65)$$

According to the described system model, the PDF of this random variable can be calculated as:

$$f_{S_n}(t) = \begin{cases} \lambda_0 e^{-\lambda_0 t}, & n = 0, \\ (\lambda_n + \delta_n)e^{-(\lambda_n+\delta_n)t}, & 0 < n < K, \\ \delta_K e^{-\delta_K t}, & n = K. \end{cases} \quad (66)$$

As shown in [18], for general distributions of relative deadline, γ_n as defined in (49) results in accurate calculation of performance measures such as job response time, in a Markovian view to the FRT system in the long run. However, since the system is not generally Markovian except for the case of exponentially distributed relative deadlines, S_n is not amenable to simple calculation. Thus, the above PDF is accurate for the exponential distribution of relative deadline and a very good approximation for the other distributions, as also confirmed by our extensive simulations.

As a prerequisite for the thermal analysis of the system (Section 5), we are interested in the sojourn-time in the idle and active modes, represented by S_I and S_A, with PDFs $f_{S_I}(t)$ and $f_{S_A}(t)$, respectively. Using (66), the former PDF can simply be obtained as

$$f_{S_I}(t) = f_{S_0}(t) = \lambda_0 e^{-\lambda_0 t}, \quad (67)$$

and the respective CDF as

$$F_{S_I}(t) = 1 - e^{-\lambda_0 t}. \quad (68)$$

However, the calculation of the latter PDF needs more discussions. Define $f_{S_{n,K}}(t)$ as the PDF of the sojourn-time in a super-state consisting of states $n, \dots, K$. Accordingly, $f_{S_{1,K}}(t)$ determines the PDF of sojourn-time in the active mode, namely $f_{S_A}(t)$. We have

$$f_{S_{n,K}}(t) = \begin{cases} f_{S_K}(t), & n = K, \\ \frac{\delta_n}{\delta_n + \lambda_n} f_{S_n}(t) + \\ \quad \frac{\lambda_n}{\delta_n + \lambda_n}\left(f_{S_n} * f_{S_{n+1,K}} * f_{S_{n,K}}\right)(t), & 0 < n < K. \end{cases} \quad (69)$$

Justification for the recursive expression for $0 < n < K$ in (69) is as follows. Suppose that the system is in state n, $0 < n < K$. The first term of the expression defines the density of staying in state n for the duration of t, and then transiting from state n to state $n - 1$ with the respective probability $\frac{\delta_n}{\delta_n + \lambda_n}$. Also, for the case of transiting in the opposite direction with probability $\frac{\lambda_n}{\delta_n + \lambda_n}$, the second term is defined as the density of staying in state n, in the super-state of states $n + 1, \dots, K$, and again in the super-state including states $n, \dots, K$ by returning back to state n, for a total duration of t.

Suppose $\mathcal{F}_{S_n}(s)$ and $\mathcal{F}_{S_{n,K}}(s)$ to be, respectively, the Laplace transforms of $f_{S_n}(t)$ and $f_{S_{n,K}}(t)$. Then, using (69) we obtain:

$$\mathcal{F}_{S_{n,K}}(s) = \begin{cases} \mathcal{F}_{S_K}(s), & n = K, \\ \frac{\delta_n \mathcal{F}_{S_n}(s)}{\delta_n + \lambda_n - \lambda_n \mathcal{F}_{S_n}(s)\,\mathcal{F}_{S_{n+1,K}}(s)}, & 0 < n < K. \end{cases} \quad (70)$$

Using (70), to calculate $\mathcal{F}_{S_{1,K}}(s)$, a polynomial fraction is obtained with a polynomial of order $K - 1$ in the numerator and one of order K in the denominator. Inverse Laplace transform of this equation leads to sum of K exponential functions with different coefficients. In other words, $f_{S_{1,K}}(t)$ has a hyper-exponential distribution. As a result, $f_{S_A}(t)$ and the respective CDF can generally be written as

$$f_{S_A}(t) = f_{S_{1,K}}(t) = \sum_{i=1}^{K} p_i e^{-y_i t}, \quad (71)$$

$$F_{S_A}(t) = F_{S_{1,K}}(t) = \sum_{i=1}^{K} \frac{p_i}{y_i}\left(1 - e^{-y_i t}\right), \quad (72)$$

where p_i and y_i are obtained by solving (70).

5 STOCHASTIC THERMAL ANALYSIS

This section follows the required steps to find the most elementary PDFs $h_{T_I}(\tau)$ and $h_{T_A}(\tau)$ (and equivalently, their CDFs) for stochastic thermal analysis of the FRT system introduced in Section 3. The approach is accurate for exponential relative deadline and approximate for the other distributions. However, for the special case of capacity 1, we provide an accurate solution for general distribution of relative deadline.

We use the following notations in this section:

$T_A^{en} \equiv$ the processor temperature at the instant of entering the active mode,

$T_A^{ex} \equiv$ the processor temperature at the instant of exiting the active mode,

$T_I^{en} \equiv$ the processor temperature at the instant of entering the idle mode,

$T_I^{ex} \equiv$ the processor temperature at the instant of exiting the idle mode. $\quad (73)$

Further, we define the conditional PDF $h_X(\tau|Y = T(t_0))$

(and equivalently the conditional CDF $H_X(\tau|Y = T(t_0))$) meant as the PDF (CDF) of X, given that the last value of Y has been $T(t_0)$, where X and Y are two random variables from those defined in (73). In the following, we first derive the PDF of temperature at the instant of entering active mode, namely $h_{T_A^{en}}(\tau)$. Next, we obtain the target PDFs $h_{T_I}(\tau)$ and $h_{T_A}(\tau)$ on the basis of $h_{T_A^{en}}(\tau)$. We assume $T_{Min} < T_{Max_H}$ in the following, as for the special case of $T_{Min} = T_{Max_H}$, the temperature is always fixed $(T(t) = T_{Min} = T_{Max_H}, \forall t)$ and the solution is trivial.

For the sake of simplicity, we describe our approach for the calculation of $h_{T_A^{en}}(\tau)$ in three steps. At first, we derive the PDF of T_A^{ex}, conditioned on the temperature at the instant of entering active mode. Then, in a similar fashion, we derive the PDF of T_I^{ex}, given the temperature at the instant of entering idle mode. Finally, we generalize the equations to obtain the (unconditional) PDF of T_A^{en}.

Calculation of $h_{T_A^{ex}}(\tau|T_A^{en} = T(t_0))$: Given $T(t_0)$ as the temperature at the instant t_0 of the most recent entrance to the active mode, we are interested in the PDF of temperature at the instant of exiting this mode, i.e. $h_{T_A^{ex}}(\tau|T_A^{en} = T(t_0))$. For this purpose, initially we calculate the corresponding CDF (see (11)):

$$H_{T_A^{ex}}(\tau|T_A^{en} = T(t_0)) = P(T_A^{ex} \le \tau | T_A^{en} = T(t_0))$$
$$= P(T_A(S_A) \le \tau), \quad (74)$$

where the CDF of S_A, the steady-state sojourn-time in the active mode, is given in (72). Using some algebra on (74), and taking into account that, in the steady-state, the temperature in the active mode is always increasing, we obtain:

$$H_{T_A^{ex}}(\tau|T_A^{en} = T(t_0)) =$$
$$\begin{cases} 0, & \tau < T(t_0), \\ P(S_A \le \Omega(\tau)) = F_{S_A}(\Omega(\tau)), & T(t_0) \le \tau < T_{Max_H}, \\ 1, & \tau \ge T_{Max_H}, \end{cases} \quad (75)$$

where

$$\Omega(\tau) = \begin{cases} -\frac{1}{N(v_A, m_L)} \ln \frac{T_{Max_L} - \tau}{T_{Max_L} - T(t_0)}, & T(t_0) \le \tau \le T_{th}, \\ \omega - \frac{1}{N(v_A, m_H)} \ln \frac{T_{Max_H} - \tau}{T_{Max_H} - T_{th}}, & T(t_0) \le T_{th} \le \tau, \\ -\frac{1}{N(v_A, m_H)} \ln \frac{T_{Max_H} - \tau}{T_{Max_H} - T(t_0)}, & T_{th} \le T(t_0) \le \tau, \end{cases} \quad (76)$$

where ω is defined in (12). In fact, $\Omega(\tau)$ is the inverse function of $T_A(t)$ in (11) and specifies the time required for the system to reach from temperature $T(t_0)$ to τ, while it is in the active mode. Taking the derivative of (75) and using (71), we obtain:

$$h_{T_A^{ex}}(\tau|T_A^{en} = T(t_0)) = \frac{dH_{T_A^{ex}}(\tau|T_A^{en} = T(t_0))}{d\tau} =$$
$$\begin{cases} d\Omega(\tau)/d\tau \, f_{S_A}(\Omega(\tau)), & \text{if } T(t_0) \le \tau < T_{Max_H}, \\ 0, & \text{otherwise,} \end{cases} \quad (77)$$

$$\begin{cases} \frac{1}{N(v_A, m_L)(T_{Max_L} - \tau)} \sum_{i=1}^{K} p_i \left(\frac{T_{Max_L} - \tau}{T_{Max_L} - T(t_0)}\right)^{\frac{y_i}{N(v_A, m_L)}}, & T(t_0) \le \tau \le T_{th}, \\ \dfrac{\sum_{i=1}^{K} p_i \left[\left(\frac{T_{Max_L} - T_{th}}{T_{Max_L} - T(t_0)}\right)^{\frac{y_i}{N(v_A, m_L)}} \left(\frac{T_{Max_H} - \tau}{T_{Max_H} - T_{th}}\right)^{\frac{y_i}{N(v_A, m_H)}}\right]}{N(v_A, m_H)(T_{Max_H} - \tau)}, & T(t_0) \le T_{th} \le \tau < T_{Max_H}, \\ \frac{1}{N(v_A, m_H)(T_{Max_H} - \tau)} \sum_{i=1}^{K} p_i \left(\frac{T_{Max_H} - \tau}{T_{Max_H} - T(t_0)}\right)^{\frac{y_i}{N(v_A, m_H)}}, & T_{th} \le T(t_0) \le \tau < T_{Max_H}, \\ 0, & \text{otherwise.} \end{cases}$$

Calculation of $h_{T_I^{ex}}(\tau|T_I^{en} = T(t_0))$: Similar to the previous step, given the temperature $T(t_0)$ of the most recent entrance to the idle mode at time t_0, and regarding that, in the steady-state, the temperature in the idle mode is always decreasing, the PDF of temperature at the instant of exiting the idle mode is obtained as:

$h_{T_I^{ex}}(\tau|T_I^{en} = T(t_0)) =$

$$
\begin{cases}
\frac{1}{N(v_I, m_L)(\tau - T_{Min})} \lambda_0 \left(\frac{\tau - T_{Min}}{T(t_0) - T_{Min}}\right)^{\frac{\lambda_0}{N(v_I, m_L)}}, & \text{if } T_{Min} < \tau \leq T(t_0), \\
0, & \text{otherwise.}
\end{cases} \quad (78)
$$

Then, obtaining the respective CDF is straightforward.

Calculation of $h_{T_A^{en}}(\tau)$: Suppose that, at time t_0, the system enters the active mode with temperature $T(t_0)$, then goes to the idle mode, and after a while returns to the active mode (or equivalently exits from the idle mode) with temperature T_I^{ex}. The conditional PDF of T_I^{ex}, $h_{T_I^{ex}}(\tau|T_A^{en} = T(t_0))$, can be obtained by combining (77) and (78) as follows:

$h_{T_I^{ex}}(\tau|T_A^{en} = T(t_0)) =$
$$
\int_0^\infty h_{T_A^{ex}}(x|T_A^{en} = T(t_0)) h_{T_I^{ex}}(\tau|T_I^{en} = x) dx. \quad (79)
$$

Using (79), we can calculate the marginal PDF $h_{T_I^{ex}}(\tau)$ as:

$$
h_{T_I^{ex}}(\tau) = \int_{T_{Min}}^{T_{Max_H}} h_{T_I^{ex}}(\tau|T_A^{en} = T(t_0)) h_{T_A^{en}}(T(t_0)) \, dT(t_0). \quad (80)
$$

On the other hand, the temperature at the instant of entering the active mode is equal to the temperature at the instant of exiting the idle mode. Therefore, $h_{T_I^{ex}}(\tau)$ in (80) can be replaced by $h_{T_A^{en}}(\tau)$:

$$
h_{T_A^{en}}(\tau) = \int_{T_{Min}}^{T_{Max_H}} h_{T_I^{ex}}(\tau|T_A^{en} = T(t_0)) h_{T_A^{en}}(T(t_0)) \, dT(t_0). \quad (81)
$$

In fact, (81) provides an integral equation which its solution gives $h_{T_A^{en}}(\tau)$. In general, this equation is not amenable to simple analytical solution. However, via well-known numerical methods, it is easy to solve the equation and obtain $h_{T_A^{en}}(\tau)$. Accordingly, we can calculate the favorite PDFs $h_{T_I}(\tau)$ and $h_{T_A}(\tau)$ as follows:

Lemma 3. *For the PDF of processor temperature in the idle mode, $h_{T_I}(\tau)$, we can write:*

$$
h_{T_I}(\tau) \equiv h_{T_A^{en}}(\tau), \quad (82)
$$

where $h_{T_A^{en}}(\tau)$ is obtained from (81).

Proof: Since job arrivals follow a state-dependent Poisson process, it is obvious from the PASTA property [42] that $h_{T_I}(\tau) \equiv h_{T_A^{en}}(\tau)$ which completes the proof. □
Then, based on (82), the calculation of $H_{T_I}(\tau)$ is straightforward.

Lemma 4. *For the processor temperature in the active mode we have:*

$$
H_{T_A}(\tau) = \quad (83)
$$
$$
\begin{cases}
0, & \tau < T_{Min}, \\
\int_{T_{Min}}^{\tau} h_{T_A^{en}}(T(t_0)) \frac{\int_0^{\Omega(\tau)} u f_{S_A}(u) du + \Omega(\tau) \int_{\Omega(\tau)}^\infty f_{S_A}(u) du}{\int_0^\infty u f_{S_A}(u) du} dT(t_0), & T_{Min} \leq \tau < T_{Max_H}, \\
1, & \tau \geq T_{Max_H},
\end{cases}
$$

$$
h_{T_A}(\tau) = \frac{d}{d\tau} H_{T_A}(\tau), \quad (84)
$$

where $\Omega(\tau)$ is defined in (76).

Proof: To obtain (83), we define
$$
H_{T_A}(\tau) = P(T_A \leq \tau) \quad (85)
$$

as the probability that, in the steady-state, the temperature in the active mode be less than or equal to τ. As the temperature is increasing in the active mode, the integration is taken in the range of $[T_{Min}, \tau]$. The integral in (83) sums up two probabilities together: 1) the probability of when the sojourn-time in the active mode is less than the relative time $\Omega(\tau)$, and thus, the temperature is lower than τ (in this case, all the sojourn-time should be accounted); 2) the probability of when the sojourn-time is more than $\Omega(\tau)$; therefore, the processor temperature exceeds τ. However, the processor temperature has been below τ for an amount of $\Omega(\tau)$ from the total duration of being in the active mode, i.e. from S_A. In this case, only the amount of time up to when the temperature reaches τ, namely $\Omega(\tau)$ should be accounted, as multiplied by the second inner integral in (83), which completes the proof. □

Using π_I and π_A from (63) and (64), and $h_{T_I}(\tau)$ and $h_{T_A}(\tau)$ from (82) and (84), $h_T(\tau)$, and consequently, $H_T(\tau)$ can be derived according to (22) and (18).

5.1 Analysis for a Specific System Setup

As mentioned before, when the system capacity is 1, we can accurately analyze the system for generally distributed relative deadlines. This analysis is started with the calculation of $h_{T_A^{en}}(\tau)$ for $K = 1$. After which, the remaining steps to derive $h_{T_I}(\tau)$ and $h_{T_A}(\tau)$ from $h_{T_A^{en}}(\tau)$ are the same as before.

Lemma 5. *For $K = 1$ and relative deadlines with a general PDF $g(t)$ (or equivalently, a general CDF $G(t)$), we have:*

$h_{T_A^{ex}}(\tau|T_A^{en} = T(t_0)) =$
$$
\begin{cases}
d\Omega(\tau)/d\tau \, e^{-\mu_A \Omega(\tau)} \left[\mu_A \left(1 - G(\Omega(\tau))\right) + g(\Omega(\tau))\right], & \text{if } T(t_0) \leq \tau < T_{Max_H} \\
0, & \text{otherwise.}
\end{cases} \quad (86)
$$
where $\Omega(\tau)$ is defined in (76).

Proof: To specify $h_{T_A^{ex}}(\tau|T_A^{en} = T(t_0))$, we only need to use the appropriate $f_{S_A}(t)$ in (77). To calculate $f_{S_A}(t)$ for the case of $K = 1$, it should be taken into account that the system leaves the active mode due to either service completion or deadline miss. Thus, to calculate the respective PDF in an accurate manner, we should consider both these cases as follows:

$$
f_{S_A}(t) = f_{V_0}(t)(1 - G(t)) + \int_t^\infty f_{V_0}(u) du \, g(t)
$$
$$
= e^{-\mu_A t}\left[\mu_A\left(1 - G(t)\right) + g(t)\right], \quad (87)
$$

where $f_{V_0}(t)$ is calculated according to (41). Substituting $f_{S_A}(t)$ from (87) into (77) completes the proof. □
The next steps to obtain $h_{T_A^{en}}(\tau)$ are the same as before, namely using (86), (78), (79), and (81).

It is worth noting that in the cases of having an approximate or accurate $f_{S_A}(t)$, also the temperature analysis would be, respectively, approximate or accurate by just following the above mentioned steps. In fact, if the job characteristics (e.g. execution-time distribution) differs from what considered in this study, in the case of obtaining $f_{S_A}(t)$ from a valid analytical or simulation method, it is sufficient to use (77), and follow the aforementioned steps to obtain $h_{T_A^{en}}(\tau)$, and consequently, $h_{T_I}(\tau)$ and $h_{T_A}(\tau)$.

TABLE 1
PARAMETERS FOR THE LINEARIZED LEAKAGE POWER

Speed	f (GHz)	v_k (v)	C_1	C_2
1	3.47	1.5	1.4038	0.0641
0.8	2.77	1.2	0.7439	0.0308
0.6	2.08	0.9	0.39	0.0077
0.4	1.38	0.6	0.204	0.0081

TABLE 2
COOLING SUBSYSTEM-RELATED PARAMETERS

Cooling mode	m_1	m_2	m_3	m_4	m_5
$CPow(m_i)$	0	0.19	0.34	0.79	2.90
$\beta(m_i)$	0.0041	0.0054	0.0059	0.0064	0.0071

6 EXPERIMENTAL EVALUATION

In this section, we evaluate some of the most elementary measures defined in Section 3. In Section 6.1, some details about the methods of computation and simulation as well as the parameter settings are presented. In Section 6.2, some elementary measures depicting the system behavior are analytically evaluated and, to show the accuracy of the method, compared against two system-level and microarchitecture-level simulations. In Section 6.3, the system behavior with respect to some other parameters and measures is evaluated to express the applicability of the proposed analytical method in finding appropriate system settings.

6.1 Experimental Setup

In this section, the details of the numerical methods used in the analysis as well as two simulation methods, namely a system-level (SL) and a microarchitecture-level (ML) simulator are discussed. The former simulator is used to investigate the accuracy of the analytical method proposed in this paper. The latter, however, is mainly used to compare the results to more realistic setups. All the experiments have been performed on an Intel Core2 Due processor of 2.53 GHz and 4 GB of RAM.

The numerical methods which are used for the desired computations in the analytical method are the rectangle integrating and Euler's differentiating methods [45] with steps of 0.01, implemented in MATLAB.

For the simulations, we have prepared a discrete-event SL simulator in Java for a stochastic FRT system with the model introduced in Section 3. It is supposed that each synthetic job consumes fixed dynamic power and the processor-related parameters have been gotten from a real processor and used in an abstract manner, as discussed in the following. The simulations have been performed for a population of one million jobs, with a confidence level of 99% within an interval of 0.01. This simulator is used to validate the analytical method for the exponential distribution and verify its accuracy for the other distributions of relative deadline.

In addition, we have prepared a ML cycle-accurate temperature- and leakage-aware thermal and power simulator by replacing the ideal processor (and abstract parameters) of the SL simulator with PTScalar [4], considering Alpha 21264 processor with the 65nm technology. For the incoming workload, we have used MiBench [47], a benchmark suite consisting of embedded programs, and truncated the selected benchmark based on the desired stochastic execution-time. Since this simulator is very time-consuming, we have run the respective simulations for only 1000 jobs, considering 10000 cycles as the tracing interval.

Table 1 shows the leakage coefficients (related to (6)) of this processor, obtained by linear approximation of the non-linear leakage power curve extracted from PTScalar results in different frequency/voltage levels. Also, according to the average dynamic power of the selected benchmark on PTScalar, $C_0' = 6.68$ is obtained. Further, similar to [4], we have considered that dynamic power in the idle mode is 25% of the active mode, i.e. we set $C_0 = 1.67$. The supply voltage used for both the idle and active modes is the same.

We have considered a cooling subsystem with five cooling modes which can be employed for DTM. The characteristics of these cooling modes are shown in Table 2, extracted from the model of a cooling fan in [46]. Further, we consider the thermal coefficient $\rho=0.0071$ ℃/W, the ambient temperature to be 45℃ and $T_{th} = 70$℃.

For the SER, we assume a linear relationship between Qcrit and temperature [28], i.e. $Q_{crit}(T) = a_1 - a_2 T$. Also,

TABLE 3

THE REQUIRED FORMULATIONS FOR SOME PERFORMANCE MEASURES OF EXPONENTIAL AND UNIFORM RELATIVE DEADLINE

	Exponential	Uniform ([a,b])
$g(t)$	$\frac{1}{\theta} e^{-\frac{1}{\theta}t}$	$\begin{cases} \dfrac{1}{b-a}, & a \leq t \leq b \\ 0, & \text{otherwise} \end{cases}$
$\Phi_n(\mu_A)$ based on (38)	$\dfrac{n!}{\prod_{k=0}^{n}(\mu_A + \frac{k}{\theta})}$	$\int_0^a t^n e^{-\mu_A t} dt + \int_a^b \left(a + \frac{1}{2}\left(\frac{t-a}{b-a}\right)(2b-a-t)\right)^n e^{-\mu_A t}dt + \left(\frac{1}{2}(a+b)\right)^n \frac{e^{-\mu_A b}}{\mu_A}$
γ_n based on (47)	$\dfrac{n}{\theta}$	$\begin{cases} 0, & n = 0 \\ n\dfrac{\Phi_{n-1}(\mu_A)^*}{\Phi_n(\mu_A)} - \mu_A, & n > 0 \end{cases}$
$f_{W_n}(t)$ based on (43)	$\dfrac{\overline{\theta}^{n-1} e^{-\mu_A t} \prod_{k=0}^{n}(\mu_A + \frac{k}{\theta})}{\mu_A(n-1)!} \left[(1 - e^{-\frac{1}{\theta}t})\right]^{n-1} e^{-\frac{1}{\theta}t}$	$\begin{cases} \dfrac{n}{\mu_A \Phi_n(\mu_A)} t^{n-1} e^{-\mu_A t}, & t < a \\ \dfrac{n}{\mu_A \Phi_n(\mu_A)} \left[t - \dfrac{(t-a)^2}{2(b-a)} \right]^{n-1} \left(\dfrac{b-t}{b-a}\right) e^{-\mu_A t}, & a < t < b \\ 0, & b < t \end{cases}$

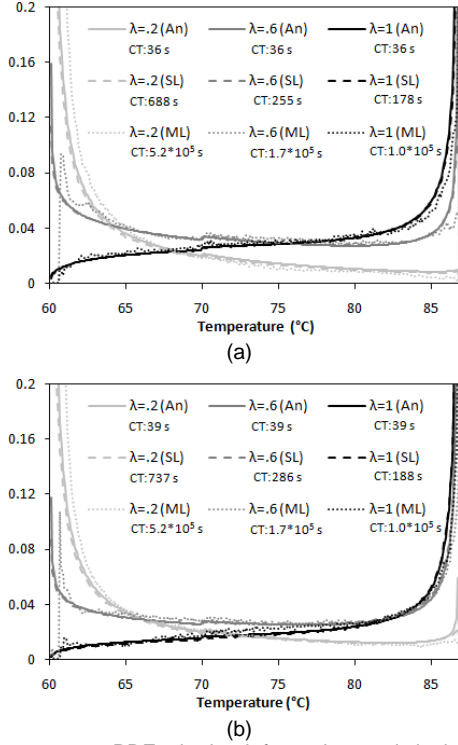* $\Phi_n(.)$ of the uniform distribution is used, as given in the immediate upper row of the table.

Fig. 2. Temperature PDF obtained from the analytical method, SL simulator, and ML simulator for (a) exponential, and (b) uniform relative deadline (the computation times (CT) of the methods are also shown).

we suppose that linear decrease in Qcrit results in exponential increase in the SER [43], i.e. $\varrho(T) = e^{-a_3 Q_{crit}(T)}$. Especially in the experiments, we assume $\varrho(T) = 0.00006e^{0.1T}$.

In our investigations, we consider the system capacity of $K=5$ and Poisson job arrivals (i.e., $\lambda_n = \lambda$ for $n = 0, \dots, 5$). Two distributions of relative deadline, namely exponential and uniform are considered. Table 3 presents the formulations of some basic performance measures for each distribution. According to these basic measures, the remaining performance-, temperature-, reliability-, and energy-related measures can be calculated by following the steps presented in Sections 4.2 and 5. The mean relative deadline for the distributions is the same, namely $\bar{\theta} = 4$. More precisely, we have considered interval [3, 5] for the uniform relative deadline. Further, to better depict the thermal behavior of the system, we consider the time unit is equal to the thermal time constant $1/\beta(m_L)$ [44] of the lumped RC model [31].

6.2 Elementary Measures and Accuracy of Analysis

In this section, we investigate the accuracy of the analytical method by comparing the analytical, SL simulator, and ML simulator results. Some complementary discussions are also included.

We begin with the most elementary temperature-related measure in the calculation of our favorite measures, namely the temperature PDF. Figs. 2(a) and 2(b), respectively, show the temperature PDF for exponential and uniform distributions of relative deadline for different values of arrival rate λ in speed $\mu_I = \mu_A = 1$. (The jump in temperature 70°C is because of switching between the cooling modes.) As seen in Fig. 2, the results of the SL simulation is

very close to the analytical method, a fact that shows the accuracy of the proposed method, even in situations that the distribution of relative deadline is non-exponential. Also, the ML simulation results reveal that, although the abstractions used in the thermal and power modeling in the analytical method leads to some deviations from a real system, the overall behavior of the system temperature has been approximated very well by the analytical method, despite the very limited population used in the respective simulations, namely 1000 jobs for each run. In fact, the errors which exist between the PDF obtained from the analytical method and the one obtained from the ML simulator have two reasons: i) the differences between the analytical and simulation boundary temperatures and the respective impacts on the overall PDFs, rooted at the existing error in the linear approximation of the actual leakage power, ii) the limited number of jobs used in each run which also affects the smoothness of the PDFs obtained from the ML simulator.

To investigate the efficiency of the analytical method, we have compared the computation time of the analytical method against the SL and ML simulation times in Fig. 2. As it is seen, while the analytical method exposes a rather small and fixed computation time, the SL simulation requires a longer time depending on the arrival rate: for smaller values of λ the fixed population (required to achieve the desired confidence level) arrives in a longer time interval. Similar justifications are valid for the ML simulation which takes much more time comparing to the previous methods, taking into account that its simulation time is reported for a population of only 1000 jobs, whereas the time for one million jobs will be almost three orders of magnitude larger.

On the temperature behavior of the system, it can be seen in Fig. 2 that increasing the arrival rate has led to spending more time in the active mode, which in turn results in higher values of the PDF in the higher temperatures. Further, it is seen that different relative deadline distributions expose different thermal behaviors. In fact, the system with exponential relative deadline generally experiences cooler situations, while the uniform relative deadline enforces a hotter processor. This is mostly due to the differences in the deadline miss rates (γ_n) of different distributions. As also shown in [6] and [18], exponential relative deadline distribution exhibits the largest deadline miss rates comparing to other distributions, resulting in more frequent visits of the idle mode. Consequently, the system is relatively cooler.

The power- and reliability-related measures, namely $\overline{Pow_{dyn}}$, $\overline{Pow_{leak}}$, $\overline{Pow_{cool}}$, and α_{se} are depicted in Fig. 3 for different arrival rates. (Since the results of SL simulator match the analytical results with very high accuracies and because of clarity of the figure, we have only shown the analytical and ML simulator results there.) It should be noted here, that the analytical and ML simulator results also match very well despite the sources of inaccuracy encountered above for the respective temperature PDFs. As dynamic power consumption in the active mode is larger than the idle mode, higher arrival rates which lead to larger sojourn-time in the active mode cause more dynamic power consumption, and thus higher
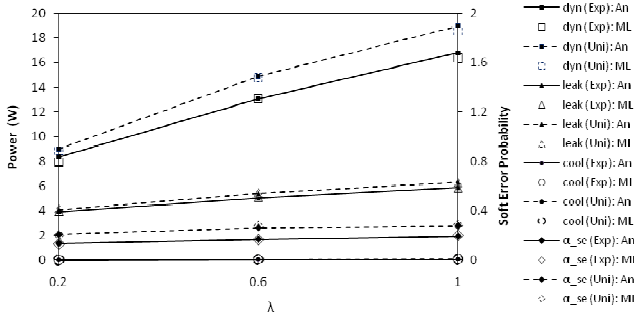
Fig. 3. $\overline{Pow_{dyn}}$, $\overline{Pow_{leak}}$, $\overline{Pow_{cool}}$, and α_{se} for exponential and uniform distributions obtained from analytical method and ML simulator.

temperatures. On the other hand, leakage power is not directly dependent on the traffic load since the parameters of the respective function in (6) are the same for both the idle and active modes (recall that we consider $\mu_I = \mu_A$). However, the leakage power depends on the temperature. As higher arrival rates lead to higher temperatures, leakage power consumption is expected to be increased in such arrival rates, a result that is seen in Fig. 3. Similar but less significant alteration is observed for cooling power and α_{se}. Also, as expected, the measures have larger quantities for uniform distribution. This result is also consistent with our previous interpretation (Fig. 2 shows higher temperatures for the unifrom distribution).

## 6.3 Application of the Analytical Method

In this subsection, we show the applicability of the proposed analytical method through studying two scenarios to obtain optimal parameter settings for different system measures.

In Fig. 4, the measures are studied for $\lambda = 1$ and different processor speeds $\mu_I = \mu_A$ between 0.4 and 1 with steps of 0.2 (the other parameters are defined in Section 6.1). As can be seen, the effect of increasing the speed on the loss probability is twofold. On one hand, increasing the processor speed diminishes α_d and α_b. On the other hand, the increased processor speed significantly raises the processor temperature, and thus, the SER, which in turn increases α_{se}. Therefore, in the determination of the processor speed for such a FRT system, there is a tradeoff between job loss due to timing problems (α_d and α_b) and soft errors (α_{se}). In fact, in our system model, this tradeoff is reflected in the total loss probability α. Fig. 4 shows that $\mu_I = \mu_A = 0.8$, among the aforementioned speeds, can be considered as the optimal service rate. Similar investigations could be performed for various arrival rates considering a constant processor speed. However, as shown in Fig. 3, the variation of λ has insignificant impacts on α_{se}. Rather, the dominant factors in the variation of loss probability with respect to λ are α_d and α_b. As these measures are temperature-independent, they are not reported in the current study (interested readers are referred to [6], [18]).

From the energy consumption point of view, we are interested in the system energy efficiency and power consumption. In fact, Δ, the measure of energy cost per successful job, reflects the overall inter-effects of the
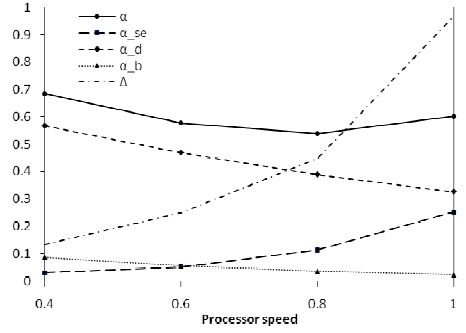
(a)
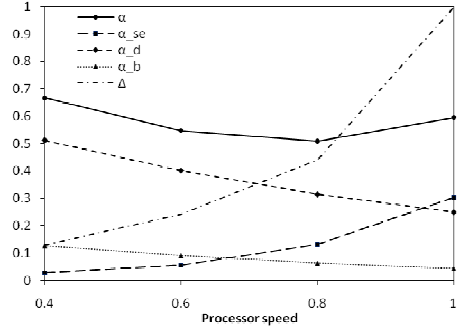
(b)

Fig. 4. Job loss probabilities (including $\alpha_d, \alpha_b, \alpha_{se}$, and α) and normalized Δ at different processor speeds ($\mu_I = \mu_A$) for (a) exponential, and (b) uniform relative deadline.

processor temperature and energy consumption, beside the system performance and reliability altogether. The results are presented in Figs. 4(a) and 4(b), respectively, for exponential and uniform distributions of relative deadline. (In the figure, the curves representing Δ are normalized with respect to the maximum value of Δ in the results which occurs for the uniform distribution.) It seems that with raising the speed, the energy efficiency degrades exponentially. As a result, it is seen that although the speed of 0.8 is optimal for the loss probability, it is not the best speed to have an energy efficient configuration.

Also, using the proposed analytical method, one can determine the best cooling modes to be selected as m_L and m_H in order to minimize the total power consumption. There exists a tradeoff between leakage and cooling powers because changing the latter in one direction results in variation of the former in the inverse direction. In this study, we have fixed $m_L = m_1$ and changed m_H as summarized in Table 4. It can be seen for the energy-related measure $\overline{Pow}$ that the best setting for both exponential and uniform relative deadlines is achieved in $m_H = m_4$.

According to the above discussions, one can use the analytical method proposed in this paper to optimally configure the FRT system through appropriate selection of different parameters with conflicting interactions, taking important mutual effects of them into account.

7 CONCLUSIONS AND FUTURE WORK

This paper considers a DTM-enabled FRT system with stochastic properties illustrated in the workload characteristics and soft errors affecting the workload. The performance, reliability, and energy consumption of the system are analytically evaluated with an emphasis on

TABLE 4
EFFECT OF VARIATION OF COOLING MODE m_H

|	m_H	m_2	m_3	m_4	m_5
α_{se}	Exp.	0.1665	0.1501	0.1360	0.1239
	Uni.	0.2401	0.2120	0.1884	0.1682
$\overline{Pow_{cool}}$	Exp.	0.1106	0.1946	0.4418	1.562
	Uni.	0.1381	0.2444	0.5593	2.0029
$\overline{Pow_{leak}}$	Exp.	5.4153	5.1169	4.8252	4.5404
	Uni.	5.8027	5.4245	5.0594	4.7005
$\overline{Pow}$	Exp.	22.3293	22.115	22.070	22.905
	Uni.	24.9225	24.650	24.600	25.685
Δ	Exp.	0.7552	0.7234	0.7021	0.7119
	Uni.	0.7583	0.7128	0.6828	0.6893

the processor thermal behavior. In fact, the mutual effects of the workload parameters, leakage and dynamic power consumption, processor temperature, mode selection of cooling subsystem, and SER are taken into account. Some important measures are calculated, including real-time performance measures, system reliability, the processor temperature PDF, leakage power, dynamic power, cooling power, and energy cost per successful job. The analytical method which is presented based on a long-run Markovian view to the system is accurate for the exponential relative deadline and approximate for the other distributions. Also, it is shown that for some specific configurations of the system, even for general distributions of relative deadline, accurate analysis is possible. Further, experiments show that the approximate method closely follows the simulation results, even for actual processors.

We believe that, with the consideration of more thermal-related processor details, the proposed analytical approach can be extended to broad system architectures (e.g., MPSoCs), even in the presence of DTM techniques.

REFERENCES

[1] W. Wang, X. Qin, and P. Mishra, "Temperature- and Energy-Constrained Scheduling in Multitasking Systems: A Model Checking Approach," *Proc. ACM/IEEE International Symp. Low Power Electronics and Design*, pp. 85-90, Aug. 2010.

[2] G. Quan and Y. Zhang, "Leakage Aware Feasibility Analysis for Temperature-Constrained Hard Real-Time Periodic Tasks," *Proc. Euromicro Conf. Real-Time Systems*, pp. 207-216, 2009.

[3] T.S. Rosing, K. Mihic and G. De Micheli, "Power and Reliability Management of SoCs," *IEEE Trans. Very Large Scale Integration Systems*, vol. 15, no. 15, pp. 391-403, Apr. 2007.

[4] W. Liao, L. He, and K. Lepak, "Temperature and Supply Voltage Aware Performance and Power Modeling at Microarchitecture Level," *IEEE Trans. Computer-Aided Design of Integrated Circuits and Systems*, vol. 24, no. 7, pp. 1042 –1053, 2005.

[5] M. Kargahi and A. Movaghar, "Performance Optimization Based on Analytical Modeling in a Real-Time System with Constrained Time/Utility Functions," *IEEE Trans. Computers*, vol. 60, no. 8, pp. 1169-1181, Aug. 2011.

[6] M. Kargahi and A. Movaghar, "A Method for Performance Analysis of Earliest-Deadline-First Scheduling Policy," *J. Supercomputing*, vol. 37, no. 2, pp. 197-222, 2006.

[7] A. Ejlali, B.M. Al-Hashimi, M. Schmitz, P. Rosinger and S.G. Miremadi, "Combined Time and Information Redundancy for SEU-Tolerance in Energy-Efficient Real-Time Systems," *IEEE Trans. Very Large Scale Integration (VLSI) Systems*, vol. 14, no. 4, pp. 323-335, Apr. 2006.

[8] L. Yuan, S. Leventhal and G. Qu, "Temperature-Aware Leakage Minimization Technique for Real-Time Systems," *Proc. IEEE/ACM International Conf. Computer-Aided Design (ICCAD '06)*, pp. 761-764, Nov. 2006.

[9] V. Chandra, R. Aitken, "Impact of Technology and Voltage Scaling on the Soft Error Susceptibility in Nanoscale CMOS," *Proc. IEEE Symp. Defect and Fault Tolerance of VLSI Systems*, pp. 114-122, 2008.

[10] T. Granlund and N. Olsson, "SEUs Induced by Thermal to High-Energy Neutrons in SRAMs," *IEEE Trans. Nuclear Science*, vol. 53, no. 6, pp. 3798-3802, Dec. 2006.

[11] C.Y. Yang, J.J. Chen, L. Thiele, and T.W. Kuo, "Energy-Efficient Real-Time Task Scheduling with Temperature-Dependent Leakage," *Proc. Conf. Design, Automation and Test in Europe*, pp. 9-14, 2010.

[12] J. Chen, S. Wang, and L. Thiele, "Proactive Speed Scheduling for Real-Time Tasks under Thermal Constraints," *Proc. IEEE Real-Time Technology and Applications Symp.*, pp.141-150, 2009.

[13] R.A. Shafik, B.M. Al-Hashimi, K. Chakrabarty, "Soft Error-Aware Design Optimization of Low Power and Time-Constrained Embedded Systems," *Proc. Conf. Design, Automation and Test in Europe*, pp. 1462-1467, 2010.

[14] Y. Liu, H. Yang, R.P. Dick, H. Wang and L. Shang, "Thermal vs Energy Optimization for DVFS-enabled Processors," *Proc. Symp. Quality Electronic Design*, pp. 204-209, 2007.

[15] S.M. Martin, K. Flautner, T. Mudge, and D. Blaauw, "Combined Dynamic Voltage Scaling and Adaptive Body Biasing for Lower Power Microprocessors under Dynamic Workloads," *Proc. IEEE/ACM Conf. Computer-Aided Design*, pp. 721-725, 2002.

[16] W.Y. Lee, "Energy-Saving DVFS Scheduling of Multiple Periodic Real-Time Tasks on Multi-core Processors," *Proc. IEEE/ACM Symp. Distributed Simulation and Real-Time Applications*, pp. 216-223, 2009.

[17] G. Hua, M. Wang, Z. Shao, H. Liu, C.J. Xue, "Real-Time Loop Scheduling with Energy Optimization via DVS and ABB for Multi-core Embedded System", *Proc. Conf. Embedded and Ubiquitous Computing*, pp. 1-12, 2007.

[18] A. Movaghar, "On Queuing with Customer Impatience until the End of Service," *J. Stochastic Models*, vol. 22, pp. 49-173, 2006.

[19] D.Y. Barrer, "Queuing with Impatient Customers and Ordered Service," *J. Operational Research*, vol. 5, no.5, pp. 650-656, 1957.

[20] L. Yan, J. Luo, N. Jha, "Joint Dynamic Voltage Scaling and Adaptive Body Biasing for Heterogeneous Distributed Real-time Embedded Systems," *IEEE Trans. Computer-Aided Design of Integrated Circuits and Systems*, vol. 24, no. 7, pp. 1030-1041, 2005.

[21] V. Devadas, H. Aydin, "On the Interplay of Voltage/Frequency Scaling and Device Power Management for Frame-Based Real-Time Embedded Applications," *IEEE Trans. Computers*, vol. 61, no. 1, pp. 31-44, 2012.

[22] M. Kargahi and A. Movaghar, "Stochastic DVS-Based Dynamic Power Management for Soft Real-Time Systems," *J. Microprocessors and Microsystems*, vol. 32, no. 3, pp. 121-144, 2008.

[23] A. Wierman, L.L.H. Andrew and A. Tang, "Stochastic Analysis of Power-Aware Scheduling," *Proc. Forty-Sixth Ann. Allerton*

Conf. Communication, Control, and Computing, pp. 1278-1283, 2008.

[24] X. Qi, D. Zhu and H. Aydin, "Global Scheduling Based Reliability-Aware Power Management for Multiprocessor Real-Time Systems," *J. Real-Time Systems*, vol. 47, no. 2, pp. 109-142, 2011.

[25] V. Chaturvedi, and G. Quan, "Leakage Conscious DVS Scheduling for Peak Temperature Minimization," *Proc. Asia and South Pacific Design Automation Conf.*, pp. 135-140, 2011.

[26] H. Huang, G. Quan, "Leakage Aware Energy Minimization for Real-Time Systems under the Maximum Temperature Constraint," *Proc. Conf. Design, Automation and Test in Europe*, pp. 479-484, 2011.

[27] M. Bagatina, S. Gerardina, A. Paccagnellaa, C. Andreanib, G. Gorinic and C.D. Frost, "Temperature Dependence of Neutron-Induced Soft Errors in SRAMs," *J. Microelectronics Reliability*, vol. 52, no. 1, pp. 289-293, 2011.

[28] I. Ukhov, M. Bao, P. Eles, and Z. Peng, "Steady-State Dynamic Temperature Analysis and Reliability Optimization for Embedded Multiprocessor Systems," *Proc. Annual Design Automation Conference,* pp. 197-204, 2012.

[29] G. Quan, and V. Chaturvedi, "Feasibility Analysis for Temperature-Constraint Hard Real-Time Periodic Tasks," *IEEE Trans. Industrial Informatics*, vol. 6, no. 3, Aug. 2010.

[30] G. Quan, S. Ren, "Leakage-Aware Real-Time Scheduling For Maximal Temperature Minimization," *IEEE Real-Time Systems Symp.*, WiP, 2010.

[31] K. Skadron, T. Abdelzaher, and M.R. Stan, "Control-Theoretic Techniques and Thermal-RC Modeling for Accurate and Localized Dynamic Thermal Management," *Proc. Symp. High-Performance Computer Architecture*, p. 17, 2002.

[32] C. Yang and A. Orailoglu, "Processor Reliability Enhancement through Compiler-Directed Register File Peak Temperature Reduction," *Proc. IEEE/IFIP Conf. Dependable Systems & Networks*, pp. 468–477, 2009.

[33] S. Murali, A. Mutapcic, D. Atienza, R. Gupta, S. Boyd, and G. De Micheli, "Temperature-Aware Processor Frequency Assignment for MPSoCs Using Convex Optimization," *Proc. IEEE/ACM Conf. Hardware/Software Codesign and System Synthesis*, pp. 111-116, 2007.

[34] H. Jung, P. Rong and M. Pedram, "Stochastic Modeling of a Thermally-Managed Multi-Core System," *Proc. Design Automation Conference*, pp. 728-733, 2008.

[35] S. Zhang and K.S. Chatha, "System-level Thermal Aware Design of Applications with Uncertain Execution Time," *Proc. IEEE/ACM Conf. Computer-Aided Design*, pp. 242-249, 2008.

[36] S. Liu, M. Qiu, W. Gao, X.-J. Tang and B. Guo, "Hybrid of Job Sequencing and DVFS for Peak Temperature Reduction with Nondeterministic Applications," *IEEE Conf. Computer and Information Technology*, pp. 1780-1787, 2010.

[37] Y. Ahn, "Real-Time Task Scheduling under Thermal Constraints," PhD dissertation, Dept. of Computer Eng., Texas A&M University, 2010.

[38] J. Rabaey, A. Chandrakasan and B. Nikolic, Digital Integrated Circuits: A Design Perspective. Prentice Hall, 2003.

[39] P.A. Ferreyra, C.A. Marques, R.T. Ferreyra, and J.P. Gaspar, "Failure Map Functions and Accelerated Mean Time to Failure Tests: New Approaches for Improving the Reliability Estimation in Systems Exposed to Single Event Upsets," *IEEE Trans. Nuclear Science*, vol. 52, no. 1, pp. 494-500, 2005.

[40] M. Chrobak, M. Hurand, J. Sgall, "Fast Algorithms for Testing Fault-Tolerance of Sequenced Jobs with Deadlines," *Proc. IEEE Real-Time Systems Symp.*, pp. 139-148, 2007.

[41] M. Kargahi and A. Movaghar, "Non-Preemptive Earliest-Deadline-First Scheduling Policy: A Performance Study," *Proc. IEEE Symp. Modeling, Analysis, and Simulation of Computer and Telecomm. Systems*, pp. 201-210, 2005.

[42] B. Melamed and W. Whitt, "On Arrivals that See Time Averages," *J. Oper. Res.*, vol. 38, no. 1, pp. 156-172, 1990.

[43] S.M. Jahinuzzaman, M. Sharifkhani and M. Sachdev, "An Analytical Model for Soft Error Critical Charge of Nanometric SRAMs," *IEEE Trans. Very Large Scale Integration (VLSI) Systems*, vol. 17 no 9, pp. 1187-1195, Sep. 2009.

[44] Y. Fu, N. Kottenstette, Y. Chen, C. Lu, X.D. Koutsoukos, H.Wang, "Feedback Thermal Control for Real-time Systems," *Proc. IEEE Real-Time and Embedded Technology and Applications Symposium*, pp. 111-120, 2010.

[45] C.F. Gerald, P.O. Wheatley, Applied Numerical Analysis. Pearson, 2004.

[46] D. Shin, S.W. Chung, E.Y. Chung, and N. Chang, "Energy-Optimal Dynamic Thermal Management: Computation and Cooling Power Co-Optimization," *IEEE Trans. Industrial Informatics*, vol. 6, no. 3, pp. 340-351, Aug 2010.

[47] M.R. Guthaus, J.S. Ringenberg, D. Ernst, T.M. Austin, T. Mudge, R.B. Brown, "MiBench: A Free, Commercially Representative Embedded Benchmark Suite," *Proc. Workshop on Workload Characterization*, pp. 3-14, 2001.
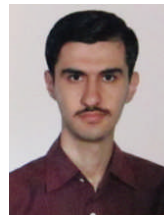
Morteza Mohaqeqi (S'11) received his B.S. and M.S. degrees in computer engineering from the University of Tehran in 2008 and 2010, respectively. He is currently a Ph.D. student of computer engineering in the Department of Electrical and Computer Engineering at the University of Tehran. His research interests include power/thermal-aware scheduling and resource management in distributed and real-time systems. He is a student member of the IEEE.

Mehdi Kargahi (M'09) received the BS degree in computer engineering from Amir-Kabir University of Technology (Tehran Poly-Techniques) in 1998 and the MS and PhD degrees in computer engineering from Sharif University of Technology in Tehran in 2001 and 2006, respectively. He is currently an assistant professor in the Department of Electrical and Computer Engineering at the University of Tehran, Iran. He has been a researcher in the School of Computer Science at the Institute for Studies in Theoretical Physics and Mathematics (IPM) from 2003. His research interests include performance modeling and power management of dependable real-time systems with stochastic properties. He is a member of the IEEE and the IEEE Computer Society.

Ali Movaghar (M'92, SM'07) is currently a professor in the Department of Computer Engineering at Sharif University of Technology in Tehran, Iran where he joined first as an assistant professor in 1993. He received his B.S. in electrical engineering from the University of Tehran in 1977 and both M.S. and Ph.D. in computer, information and control engineering from the University of Michigan at Ann Arbor in 1979 and 1985, respectively. He visited INRIA in France in 1984, worked at AT&T Laboratories during 1985-1986, and taught at the University of Michigan during 1987-1989. His main areas of interest are performance and dependability modeling, verification and validation, computer networks, and distributed real-time systems. He is a senior member of the IEEE.